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TECHNICAL MEMORANDUMS

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

No. 903

EFFECT OF TRANSITION IN CROSS-SECTIONAL SHAPE ON THE

DEVELOPMENT OF THE VELOCITY AND PRESSURE DISTRIBUTION OF

TURBULENT FLOW IN PIPES

By Edwin Mayer

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I. INTRODUCTION

Very many flow processes of engineering application occur within regions where the cross sections may vary not only in area but also in shape. In the most general case the flow channel will also possess curvature so that essentially there are three factors that may be expected to affect the velocity and pressure distribution:

1. Change in the side of the flow cross section leading to a mean acceleration or retardation.

2. Curvature.

3. Change in the shape of the cross section.

Examples of flow in which these three influences operate to different degree are those in the curved intake pipes of turbines and in the impeller and guide wheels of pumps and compressors of all kinds. In order to gain insight into the difficult processes that occur, each of the three influences above must be investigated. So far, this has been done for those enumerated under 1 and 2 above. Thus Nikuradse (reference 1) and Dönch (reference 2) considered the velocity and pressure distribution in convergent and divergent channels and others, particularly Nippert (reference 3) considered curved channels of constant cross-section shape. With regard to the third factor, namely, the change in shape of the cross section

while the area remains constant, no investigation results
are as yet available.* Such an investigation will be the
subject of the present paper. For this purpose it is ne-
cessary to consider the velocity and pressure relations
over each entire cross section so that we are confronted
with a three-dimensional problem.

II. THE TEST SET-UP

One of the most common types of transition in cross-
sectional shape is that of a circle into a rectangle.
Since, moreover, the built-up velocity distributions differ
considerably in the flow through circular and rectangular
cross sections, it may be expected that the effect of the
change in cross section on the velocity and pressure rela-
tions will show up very strongly. For this reason, there
is considered in this paper the flow where the shape of
cross section changes from circle to rectangle or in the
reverse direction, from rectangle to circle. The tests
were carried out with air as the medium.

Figure 1 shows the test set-up for the investigation
of the velocity and pressure distributions in the transi-
tion from a circular cross section (diameter, 190 mm) into
a rectangular one of the same area (width, 238 mm; height,
119 mm). These relatively large cross-sectional areas
were chosen in order that the percent change in cross-
section area resulting from the insertion of the velocity-
and pressure-measuring apparatus should be as small as
possible and thus the actual velocity and pressure distri-
butions measured with the greatest possible accuracy. The
tests were performed in the open air. With the intake
chamber k not connected to the pipe the readings of the
micromanometers which served to measure the velocity and the
pressure were subject to rather large irregular fluctuations
on some days as a result of wind impact on the open pipe.
In order to eliminate this difficulty in the reading of the
instruments, an intake chamber k of approximately 2 cubic
meters volume was joined to the end of the pipe. The bottom
of the chamber is formed of two sheet-metal layers l hav-

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*One of the few suggestions for the necessity of taking in-
to account the effects of change in cross-sectional shape
on the development of the velocity distribution is found in
the dissertation of E. Schröder (reference 4).

ing orifices through which the air is sucked in, the
chamber being otherwise completely closed. To avoid dis-
turbances resulting from jet contraction and formation
of vortices at the sharp pipe edges at the inlet to the
entry run of pipe a, an entry throat e is inserted
between the intake chamber and the circular pipe through
which throat the air in the intake chamber is gradually
accelerated.

     In the preliminary tests, it was found that the ve-
locity profile at the entry run of pipe was asymmetrical
to the pipe axis, more air flowing through the upper or
lower part of the circular section, depending on the dis-
tance of the profile from the pipe inlet. In order to
avoid this, two honeycombs g, one directly behind the
throat and the other 5 meters distant, were installed in
the pipe a. They consist of thin-walled sheet metal
tubes of about 250-millimeter length and 25-millimeter di-
ameter arranged to fill out the cross section by honey-
combs. The installation of these resulted in symmetric
velocity distribution with respect to the pipe axis.

     The entry run of pipe was 17 meters long, that is,
about 90 diameters. The pipe is formed of an 8-meter
length of normally welded sheet metal while the remaining
length of 9 meters was of seamless, drawn sheet brass.

     Preliminary tests were at first necessary in order to
see whether the entry run of 90 diameters was sufficient
to give at its end a built-up turbulent velocity profile.
The condition for this stable velocity profile is that it
no longer changes with increase in entry length. Accord-
ing to investigations by Kirsten (reference 5), the length
of pipe chosen is sufficient to give a stationary, turbu-
lent velocity profile. In the present case, because of the
asymmetrical entry, the possibility arose that these dis-
turbances might require a longer entry run of pipe.

     An accurate velocity profile was therefore taken at
the end of the entry run, that is, after a length of 90 d
and also one after a length of 80 d. Comparison gave an
accurate agreement between the two velocity distributions.
A further proof that the length of entry run was sufficient
will be provided in the more detailed discussion below of
the initial profile (section VI). It is essential to have
a stationary velocity profile at the end of the entry run
in order to obtain independently the effect of the change
in cross section on the velocity and pressure distributions

and avoid disturbances that may be ascribed merely to a short length of pipe. It must be noted, however, that the requirement of a built-up turbulent velocity profile at the beginning of the transition section very seldom is met with in practice where such transitions in cross sections occur. It is necessary, however, to assume such built-up profiles in systematic tests in order to be able to simplify to some extent the difficult relations involved. Only then will it be possible to consider measurements with other entry profiles that occur more in practice.

At the end of the entry run of pipe, there is the transition piece v which permits the transition from the circular cross section to the rectangular while maintaining the area the same. In order to be able to follow further the development of the velocity and pressure distribution after the exit of the air from the transition piece and to eliminate disturbing reaction effects on the velocity and pressure distribution in the transition piece itself the air after leaving the transition piece flows into a 5 meters long rectangular exit run b. The construction of the latter may be seen from figure 2 which shows a section of it. It is assembled from 3 millimeters thick smooth sheet brass and resid on the cast iron frames r which are screwed on at a distance of 1 meter apart on two U beams s. In order to obtain sharp edges the two latter sheets sb are welded to the base plate bb and screwed together with the cover plate db. The exit run b ends in a deflecting chamber u (fig. 1) out of which the air is sucked by a blower.

The test set-up for the flow in the opposite direction where the cross section changes from rectangular to circular of equal area is shown in figure 3. The entry run of pipe a connected ahead of the transition piece v has a length of 11 m = 70 d_h (d_h = hydraulic diameter $= \dfrac{4F}{U} = 0.1586$ m, F = area of section, U = perimeter of cross section), whereas the connecting exit run b of circular section is again 5 meters long. In order to avoid disturbances, there is in this case, too, at the entrance to the entry run an entry throat e.

In the case of the flow direction rectangle $\longrightarrow$ circle there is also to be invested the development of the velocity distribution along the entry run itself, no measurements of this kind having so far been published according to the author's knowledge. In order to carry out

these measurements it was necessary to dispense with the intake chamber ahead of the entry throat since because of the deflection of the intake air this chamber would have made the velocity profile in the first part of the entry run asymmetrical so that the development of the velocity profiles could not be followed. Similarly, it was also necessary to dispense with a honeycomb in the entry length of pipe since it would give rise in part of the pipe to a velocity distribution that would not agree with that of the same section without the honeycomb. It was all the easier to dispense with the honeycomb as it was found that in the initial rectangular run of pipe there appeared no tendency toward a velocity distribution asymmetrical with respect to the axis. On account of the absence of the intake chamber, the tests for the flow direction rectangle ⟶ circle were carried out only on calm days.

As in the case of the transition circle ⟶ rectangle it also was established for the reverse direction whether the chosen length of pipe was sufficient for attaining a fully built-up turbulent velocity distribution. As may be seen from figure 7, a stationary built-up turbulent velocity distribution is attained at the very entrance to the transition piece, so that the length of the initial run of pipe is sufficient.

The remaining construction of the test set-up corresponds to the transition circle ⟶ rectangle. The air flows out of the transition piece v through the circular exit run b into the deflecting chamber u out of which it is sucked by the blower and is again discharged into the free atmosphere. The purpose of the Prandtl tube p indicated in figures 1 and 3 will be discussed in section V.

III. THE VELOCITY AND PRESSURE-MEASURING APPARATUS

For the measurement of the velocity and pressure distributions in the different cross sections a pressure sphere whose construction and principal dimensions may be seen from figure 4 was made use of. The entire apparatus rests on a base plate g which is screwed on to the pipe over the measuring orifices (s in fig. 8) to be further described below. (The manner of this attachment may also be seen from fig. 11.) On the base plate g is a cylinder w rotatable in a plane in both directions and to this is attached the support h of the shaft of the sphere.

The magnitude of the angle of rotation $\pm\psi$ of the cylinder with respect to the vertical position is read off on a graduated circle m. The shaft s of the sphere is axially displaceable in the support and can be turned about its axis (angle of rotation α). At the upper end of the support is the headpiece o rotatable in the support, ground smooth on one side and provided with a vernier scale n. Through this the sphere shaft, which is provided with a mm scale along its length is visible so that the depth of lowering of the pressure sphere into the transition section may be determined. At the upper end of the shaft there is a disk r rotating with it and provided with a graduated circle on which the angle of rotation $\pm\alpha$ of the shaft with respect to a determined zero position may be read off by rotating the pointer z over the disk until the reflection of the tip of the pointer coincides with the pointer tip z_1 over the mirror sp.

By adjusting the angle ψ and the lowering depth, all points of the lower half and a portion of the points of the upper half of each section of the transition piece may be reached. The sphere k itself has four pressure orifices (fig. 5a) in contrast to the spheres that have so far been employed in the Karlsruhe Laboratory for flow machines, where 12-millimeter diameter spheres provided with five pressure orifices are employed. (fig. 5b). A detailed description of a pressure sphere with five orifices may be found in the paper by Krisam (reference 6) so that an explanation of the principle of the measurement of velocity and pressure with the aid of the sphere may be dispensed with here. A deviation was made from the usual type of construction in order that a smaller sphere diameter may be obtained. With the sphere diameter chosen of 8 millimeters, it is no longer possible to weld securely and air tight the five small pressure tubes required for a sphere with five orifices (0.8 mm outer diameter, and 0.5 mm inner diameter) that serve to convey the pressure through the sphere shaft. In order to keep the diameter of the sphere down to 8 millimeters and at the same time to maintain the dimensions of the pressure tubes, there is no choice than to attempt to construct a practical instrument with fewer than five orifices which should permit the measurement of the velocity in magnitude and direction as well as of the pressure. In using the 8-millimeter sphere with four orifices, it is necessary, because of the elimination orifice 3 (fig. 5b) to define the coefficient k_δ for the determination of the angle δ (angle between the velocity direction and the perpendicular to the sphere shaft in the

meridian plane which contains the orifices 1 and 2) differently than that for the sphere with five orifices. In our case

$$k_\delta = \frac{k_2 - k_1}{k_2 - k_4} = \frac{h_2 - h_1}{h_2 - h_4}$$

where k_1, k_2, and k_4 are nondimensional coefficients whose values must be determined by calibration from the relation

$$k_{1,2,4} = (h_{1,2,4} - p/\gamma) \, 2 \, g/c^2$$

as a function of δ. (h_1, h_2, h_4 are the pressure heads at orifices 1, 2, and 4; p is static pressure; γ, specific weight of the flow medium; g, acceleration of gravity; c, velocity of flow.) The k_δ curve plotted against $+\delta$ and $-\delta$ must be of such a form that to each value of k_δ there corresponds a definite angle and not several values of δ. In order to satisfy this condition, the orifice 2 on the sphere was somewhat inclined toward the shaft so that the axis of the orifice forms an angle of $6°$ with the middle plane of the sphere at right angles to the shaft axis. For the same reason, the orifice 1 assumed a somewhat different position, its axis forming with the middle plane an angle of $30°$ (fig. 5a). In this way there was obtained a unique curve for k_δ as a function of the angle $\pm\delta$ as is shown in the calibration curve of figure 6 for the values of δ between $+35°$ and $-35°$. For values of δ larger than $\pm35°$, the k_δ curve becomes many-valued. In this range therefore, the pressure sphere is no longer applicable. In the case of the tests under consideration, however, no such large values of δ were to be expected.

In the measurement of the velocity and the pressure at any position with the aid of the pressure sphere apparatus, the procedure is as follows. After adjustment of the sphere coordinates of the corresponding measuring station, that is, the lowering depth of the shaft and the angle $\pm\psi$, that sphere is rotated until the pressure at orifice 4 becomes equal to that at orifice 5. On the scale attached to the sphere shaft, this angle of rotation $\pm\alpha$ can then be read off. The zero line for measuring the angle α lies on the line of intersection of the plane of the disk and the meridian plane of the sphere through the orifices 1 and 2. After the determination of the value of k_δ

from the manometer readings h_1, h_2, and h_4 by the re-
lation given above, there is determined from the calibra-
tion curve the angle $\pm\delta$ corresponding to this value of
k_δ. There are furthermore obtained from the calibration
curve the velocity coefficients k_1 and k_2. The speed
and pressure energy is then obtained from the relations

$$\frac{c^2}{2g} = \frac{h_2 - h_1}{k_2 - k_1} \quad \text{and} \quad \frac{p}{\gamma} = h_1 - k_1 \frac{c^2}{2g}$$

The velocity c thus determined is split up into three
components c_x, c_y, and c_z at right angles to one an-
other. The c_z component is parallel to the longitudinal
axis of the test length and is considered positive in the
direction of the main flow. In the plane at right angles
to it are the c_x and c_y axes. Their positive sense is
taken in the usual manner so that the rotation from the
positive c_x axis to the positive c_y axis gives a right
hand screw direction to the positive c_z axis. From the
measured angles α, δ, ψ, and the determined velocity c,
the velocity components are computed from the relations

$$c_x = c(\cos\delta \sin\alpha \cos\psi - \sin\delta \sin\psi),$$

$$c_y = -c(\sin\delta \cos\psi + \cos\delta \sin\alpha \sin\psi)$$

$$c_z = c \cos\delta \cos\alpha$$

Concerning the effect of the shaft of the sphere on
the velocity measurement, the following may be remarked.
In the most unfavorable case, that is, in the measurement
of the velocity at the lowest measuring positions of the
circular cross section of the transition pieces, the 9-
millimeter diameter shaft obstructs the cross section over
a length of approximately 180 millimeters. This means
therefore a reduction in area of the total section of about
16.2 cm^2, which is 5.6 percent. The sphere was calibrated
by the method described in detail in the paper by Krisam
(reference 6) and also for a circular pipe of 190-millimeter
diameter, the shaft extending 55 millimeters into the cross
section. The comparison velocity and comparison pressure
were measured with a Prandtl tube whose shaft extended di-
ametrically opposite the sphere also 55 millimeters into
the cross section. The distances of the pressure slot and
the dynamic pressure orifice from the shaft of the Prandtl
tube correspond to that required in the Handbuch der Ex-
perimental Physik (reference 7). The dimensions there in-

dicated are such that the effect of tube shaft on the measurement of the dynamic and static pressures is eliminated and there may thus be obtained the true velocity at a point. It is of importance to see that in the calibration of the sphere apparatus there should be a similar decrease in area by 10 cm² = 3.5 percent. The effect of the sphere shaft on the velocity measurement therefore involves in the most unfavorable case a decrease in area of 6.2 cm². In order to test the effect of this the velocities in the pure circular cross ection were measured first with the sphere and then with a small pitot tube of 2 mm shaft diameter. The two velocity distributions thus obtained show a very good agreement from which it may be concluded that the effect of the sphere shaft on the velocity measurement is negligible. Also the pressure measurement carried out with the aid of the sphere gave for each point of the circular cross section the same pressure and hence led to the conclusion that the shaft of the sphere is also without effect on the pressure measurement. As mentioned in the paper by Krisam the sphere gives erroneous values in the neighborhood of the wall. The velocities and pressures are measured too high there and the measurements of the angles also disagree.

The causes for these errors are first, the fact that the velocity distribution in the neighborhood of the wall is disturbed by the sphere and particularly the fact that the sphere is located in a region of large velocity drop, so that very different velocities are found at each of the orifices of the sphere. At such points therefore the velocities were measured with a small pitot tube (fig. 7), the static pressure being taken at the same time at the pressure orifices at the wall. Also in the measurement of the velocity in the neighborhood of the wall in those cross sections in which the pressure varied over the periphery of the cross section of the static pressure of the wall orifice was differentially connected to the dynamic pressure at the other end of the micromanometer and then the reading measured with the sphere instrument, corrected to the corresponding actual static pressure at the point under consideration. In the measurement with the pitot tube, the latter was placed in the support in the position of the sphere shaft. In order to be able to measure the velocity in the same plane, the pitot tube was bent in the form of a hook. With this instrument, it is possible to take velocity measurements at points having a minimum distance from the wall of 1-1/2 mm. As an indication of the accuracy of the velocity measruements, it may be said that the discharge V given by the continuity law V =

$F c_{zm}$ (c_{zm} is mean velocity through section) at the various measuring sections F gave the same value with a deviation of ± 0.75 percent.

IV. THE TRANSITION PIECES

Two transition pieces were investigated. Both have the following principal dimensions as may also be seen from figures 8 and 9:

Diameter of circular section: 190 mm

Rectangular section: 238 × 119 mm²

The transition length, that is, the distance between the fully built-up circular and rectangular sections, amounts in the case of the first transition piece I to 440 millimeters and in transition piece II to 110 millimeters.

A circle may be deformed into a rectangle of the same area along a straight axis in an infinite number of ways. From the condition, however, that there is a flow medium through the sections the number of possible deformations becomes strongly limited. The main condition to be satisfied by the transition piece is that the transition be uniform and continuous along the entire length, that is, that it follow a definite law of deformation, so that there be no discontinuities in the transition from one section to the other and each intermediate section possess the same area.

The transition length for piece I is so chosen that the angles between the axis and the walls of the transition piece remain below 5°. On the other hand, the angles of transition piece II, on account of the shorter length, may reach a value up to 19°. The latter transition piece was mostly for the purpose of investigating whether, because of local negative accelerations, separation from the wall occurs in spite of a constant mean velocity. The law of deformation according to which the circle is transformed into the rectangle will be explained with the aid of figure 10.

There are first shown the initial and end cross sections. It is readily seen that the vertically hatched areas 2A subtracted from the circle are equal to the hori-

horizontally hatched areas 2B added to the rectangle.
Seven intermediate cross sections are indicated for form-
ing the transition piece. The contours of these sections
were obtained in the following manner. Areas A and B
were divided into eight equal partial areas as shown in
the figure. The lines joining the corresponding division
points give the contour of an intermediate cross section
(fig. 10, where this construction is shown for the middle
cross section 5). The partial areas subtracted at the
upper and lower sides are added laterally to the circle,
so that each intermediate section has the same area as the
initial and end section of the transition piece. The sec-
tions finally obtained are shown in figure 8 with the only
difference that the edge arising at E is somewhat broken,
the areas, of course, still remaining the same.

The transition pieces were cast of silumin since the
latter gives a very smooth surface and retains its shape
very well. Both of these properties must unconditionally
be required of the material used for the casting since it
is subsequently impossible to work the inner surfaces of
the transition piece, the surfaces being merely smoothed
over with glass paper. On the upper sides of the transi-
tion pieces are cast the seats a (figs. 8 and 9) to which
the velocity apparatus may be screwed on and extended
through the milled slots s at the various cross sections.
The slots of those cross sections at which no measurements
were taken were made airtight by accurately cast lead cov-
ers. At the ends of the transition pieces themselves are
the inital and end cross sections 200 millimeters apart, so
that transition piece I has a total length of 400 + 2 × 200 =
840 millimeters and piece II a length of 110 + 2 × 200 =
510 millimeters. Transition piece I is provided with 11
measuring sections and hence with 11 seats with slots,
whereas transition piece II possesses five measuring sec-
tions. Laterally and at the under sides there are located
"eyes" au at each second section of piece I and at each
section of piece II, which are provided with orifices b
and tubes t for taking the static pressure in measuring
the velocities in the neighborhood of the wall with the aid
of the pitot tube.

It was first necessary to test the cast transition
pieces for their suitability for the tests under consider-
ation. It was particularly necessary to investigate
whether the required cross sections agreed with those of
the actual transition pieces and whether the area was con-
stant as required. For this purpose each cross section of

the transition pieces was measured and plotted with the
aid of a caliper instrument. This instrument consisted
of a pointed metal rod st which is inserted in the
support h of figure 4 in place of the sphere shaft s,
figure 11. It is then possible to measure the lower and
a portion of the lateral parts of the cross sections ac-
curately by reading the angle with the scale m and the
depth of insertion of the rod. To measure the portion of
the cross sections not reached with the straight rod,
there is provided a hooked point sp as shown in figure
11. The areas of the sections thus measured out and plot-
ted were obtained by a planimeter and are given in tables
1 and 2. The cross sections of the circular and rectan-
gular entry and exit runs all had an area of 283.0 cm².
As may be seen from the above tables the maximum devia-
tion from the required area (F = 283.0 cm²) was about
5 cm², which corresponds to an error of 1.8 percent. These
inaccuracies must be reckoned with since hardly any greater
accuracy could be expected with the pieces thus roughly
cast and not worked over.

5. THE TEST PROCEDURE

Corresponding to the normal output of the blower,
the tests for the two transition pieces investigated were
carried out for a Reynolds Number of about R = 192,000,
the latter being computed in the case of the transition
circle ⟶ rectangle for the fully built-up circular
cross section with diameter d = 0.190 m and for the
transition rectangle ⟶ circle for the fully built-up
rectangular cross section with width b = 0.238 m and
height h = 0.119 m. Corresponding to this circle diam-
eter and the hydraulic diameter d_h computed from the
relation $d_h = 4\ F/U = 0.1586$ m (F = area and U = wetted
perimeter of the section) there is obtained for approxi-
mately equal Reynolds Number $R = c_{z_n}\ d/\nu$ or $R =$
$c_{z_n}\ d_h/\nu = 192,000$ for the transition circle ⟶ rec-
tangle a mean air velocity of $c_{z_n} = 14.0$ m/s and for
the transition: rectangle ⟶ circle $c_{z_n} = 16.3$ m/s.
The kinematic viscosity ν of the air was assumed to be
13.6×10^{-6} m²/s.

In order to have a measure for the quantity of air

discharged there is a Prandtl tube p located at the
sections shown in figures 1 and 3 in the pipe axis. It
was found by velocity measurements in section 10 that
this apparatus for normal discharge of air for the case
of the transition circle ⟶ rectangle shows a velocity
of 17 m/s and for the transition rectangle ⟶ circle a
velocity of 20.3 m/s. In view of the small size of the
direct current rectifying network from which the motor
driving the blower was fed, the rotational speed depended
on the loading network, so that at irregular time inter-
vals different velocities with different discharge quan-
tities were obtained which remained constant for some
length of time. In order therefore to obtain a constant
air flow for determining the velocity and pressure dis-
tribution in a section of the transition piece, it would
have been necessary to vary the field of the motor by a
resistance to correspond with the change in rotational
speed. In order to avoid the need for this continuous
regulation, the velocity and pressure measurements were
carried out by the following method.

In measuring the velocity c and the pressure p
at any point with the aid of the pressure sphere the ve-
locity c_{st} was recorded with the aid of the Prandtl
comparison instrument simultaneously with the three micro-
manometers giving the pressure heads at the orifices 1,
2, and 4. A ratio c/c_{st} may be determined directly from
this measurement. By checking, it was found that this
ratio c/c_{st} gives for a definite measuring point a value
which is constant for discharge quantities that are about
10 percent higher or lower than the normal discharge quan-
tity. Similar velocity distributions were thus obtained
for all point at which the velocity was measured with the
sphere. Also, for the points in the neighborhood of the
wall where the velocity was obtained with a pitot tube
c/c_{st} was obtained as a constant ratio for the same
point, so that it may be said that the velocity profiles
at the measuring planes of each cross section remained
similar in spite of the change in the Reynolds Number in
the given region. To determine the velocity distribution
at the various cross sections, the ratio c^2/c_{st}^2 or,
taking the square root, c/c_{st}, was therefore directly
determined from the monometer readings for the measuring
point under consideration. From this was obtained the
absolute value of the velocity for constant mean velocity
c_{z_m} for the transition circle ⟶ rectangle, according
to the relation $c = 17\, c/c_{st}$; for the transition rectan-
gle ⟶ circle the relation is $c = 20.3\, c/c_{st}$.

For determining the pressures, three pressure and velocity measurements for three different discharge quantities were taken at each measuring point. The speed of the motor was so regulated that the blower first drew in the normal quantity of air and then quantities 10 percent higher and lower, respectively. Corresponding to the values of $c_{st}^2/2\ g$ in mm water, the pressure heads were measured with the sphere and graphically plotted. This gave for all measuring points a straight line. Taking into account the atmospheric pressure prevailing on the day of the test and the moisture of the air and its temperature, the reading corresponding to the mormal air quantity for $c_{st}^2/2\ g$ in mm water was computed from the following relations. In the transition circle $\longrightarrow$ rectangle, the velocity head of the tube corresponding to the normal discharge $c_{st}^2/2\ g = 17^2/2\ g = 14.75$ m air column.* This value, recomputed from the relation

$$c_{st}^2/2\ g = 14.75 \times 1000\ \frac{\gamma_L}{\gamma_W}\quad\text{gives the required dynamic}$$

pressure for that particular day at the Prandtl tube in mm water for the normal quantity of air flowing through the pipe (γ_L = specific weight of the air, γ_W = specific weight of the water). It is then possible to obtain the pressure in mm water from the graph. This pressure must be converted in meters of air from the relation $p/\gamma_L =$

$$p/\gamma_W/\frac{\gamma_W}{\gamma_L\ 1000}\quad\text{since only the pressure head expressed in}$$

meters of air column measured at the same point on days with different γ_L gives a constant value. The same computation procedure was applied to the transition rectangle $\longrightarrow$ circle for a velocity head $c_{st}^2/2\ g = 20.3^2/2\ g = 21.1$ m air column for the determination of the pressure. This method of measuring the pressure has yet another advantage in that it provides for each measuring point a mean value from the three readings for the determination of the angles and the ratio c/c_{st} and so the constantly occurring small reading errors cancel.

With regard to the location of the measuring points at each of the cross sections, it is to be observed that

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*For these small pressures, it is convenient to indicate the data in meters air column.

they all lie on parallels to two perpendicular axes. The
axes are the lines of symmetry of each cross section, their
position being determined from the cross section contours
found with the aid of the caliper apparatus. The number
of measuring points was chosen in such a manner that at
positions with rather flat velocity profiles, that is,
small velocity drops, the distance between the measuring
points amounted from 1.5 to 2 mm. In those parts of the
cross sections where secondary flows were to be expected,
the distance between the points was made smaller. As an
example, figure 12 gives the arrangement for the lower
halves of sections 0 and 10.

With regard to the numbering of the straight lines
parallel and at right angles to the axes, it is to be ob-
served that at each cross section lines having the same
distance from the axes are assigned the same number and
therefore all lie in the same measuring plane which is
parallel to the principal axis of the transition piece and
at right angles to one of the two axes of each cross sec-
tion. The letter a following a number indicates that
this line is 5 mm distant from the line having that number.
Thus, for example, the measuring line 1a is 5 mm from the
line 1. The letter b indicates a distance of 10 mm and
c   a distance of 15 mm. Lines with two successive numbers
and no letters have a distance of 20 mm apart.

In the tests the result was found that in each sec-
tion both for the transition circle ⟶ rectangle and
rectangle ⟶ circle the velocities occurring at all at-
tainable measuring points were exactly as large as at those
symmetrically located with respect to them in another quad-
rant of the cross section. Only at the measuring points
very near the location of the measuring apparatus did small
deviations appear. Also with respect to the velocity di-
rections no essential differences could be established in
the various quadrants of the cross sections. In several
intermediate cross sections therefore, only the velocity
and pressure distribution in one fourth of the cross sec-
tion was measured and the symmetry checked by random tests.
For this reason, in presenting the test results in the fol-
lowing sections, only the velocity distributions for a
fourth of the cross section will be given.

VI. THE INITIAL PROFILE IN CROSS SECTION 10 OF THE

TRANSITION PIECE FOR THE FLOW DIRECTION

CIRCLE ⟶ RECTANGLE

The velocity profile for the circular cross section 10 in the flow-direction circle ⟶ rectangle denoted in what follows by C ⟶ R(I) and C ⟶ R(II) is the same for all radial planes of the two pieces and thus possesses rotational symmetry. The profile is shown in figure 13. It was proven that this profile represents the fully developed turbulent condition. In order to make a comparison with the measurements of other authors, the velocity distribution was checked, using the Prandtl mixing length relation (reference 8)

$$l = \sqrt{\frac{\tau}{\rho} \Big/ \frac{dc_z}{d\eta}}$$

(l, mixing length; $\rho = \gamma/g$, density). Taking into account the linear drop of the shear stress τ as a function of the distance η from the wall according to the relation

$$\frac{\tau}{\tau_0} = \frac{r - \eta}{r}$$

(τ_0, shear stress at the wall; r, pipe radius) and substituting the pressure drop in the direction z of the pipe axis into the relation

$$\tau_0 = \frac{r}{2} \frac{dp}{dz}$$

there is obtained the distribution of the mixing length shown in figure 14. For the distribution of the mixing length, Prandtl gives the following formula

$$\frac{l}{r} = 0.14 - 0.08 \left(1 - \frac{\eta}{r}\right)^2 - 0.06 \left(1 - \frac{\eta}{r}\right)^4$$

In comparing the values of l/r, computed from the above formula with those found from the measurement, very good agreement is obtained.

If it is desired to give the velocity distribution directly by means of an exponential law of the form $c_z/c_{st} = a \eta^{1/n}$ (a is constant; η, distance from the wall; $1/n$, exponent), the logarithm of c_z/c_{st} is plotted against the distance from the wall as abscissa and the exponent $1/n$ found, in the usual way, as shown in figure 15. (In order to obtain positive values, the arguments of the ordinates are multiplied by 10.) From the slope of the straight line in figure 15, which is drawn from the initial profile of figure 13, the value of the exponent may be directly obtained as $1/n = 1/8.3$. This value, too, agrees well with the measurements of other authors.

That both the mixing-length distribution and the exponential law agree with the measurements of other experimenters is another confirmation of the fact that at the entrance to the circular cross section of the transition piece, the turbulent velocity profile is fully built up. In addition, the conclusion may be drawn that the transition has no effect on the velocity distribution in section 10.

VII. THE FLOW IN THE ENTRY RUN OF RECTANGULAR SECTION

As already mentioned, in the case of the transition rectangle $\longrightarrow$ circle, denoted in what follows by R $\longrightarrow$ C(I) and R $\longrightarrow$ C(II), it was first necessary to establish whether at the entrance to the transition piece there was a fully built-up turbulent velocity distribution. For this purpose it was necessary to obtain the velocity distribution in several cross sections of the entry run of pipe in order to see from the comparison whether and where a position occurred from which position on the velocity distribution over the entire cross section no longer varied, that is, whether the initial run of pipe was long enough. For the sake of completeness and in view of the lack of such measurements in entry runs of rectangular cross section, the velocity profiles were measured along the entire length of the run at the sections indicated in figure 16, the measuring apparatus used being the pitot tube of figure 7. The attempt to use the sphere apparatus encountered difficulties since the fluctuations of the fluid columns in the three micromanometers, particularly in the measurement of the cross sections with short entry runs, were so large that the readings became too inaccurate.

As a result of the difficulties, considered more in detail in section VIII, which arose in following the development of the velocities in a spatial flow, it was found best also in this case to present the conditions in the various cross sections in the form of "isotachs" (c_z/c_{st} or c_z/c_{max} = constant). The measured velocity profiles in cross section 0 may be seen on figure 17 whereas the isotach diagrams for all cross sections indicated in figure 16 are given in figures 18 to 26.

It may be seen that at cross section I nearest the entrance (fig. 18) there is a sufficiently uniform velocity distribution over the entire cross section with a rapid drop toward the wall as may also be expected by analogy with the tests of Kirsten (reference 5) in entry runs of circular cross section. For after such a short path of 1.55 m = 9.5 d_h in the pipe the particles of air further removed from the bounding walls are not yet affected by the shear stress in the neighborhood of the wall. In cross section II (fig. 19) there will be found, though only in a small region in the direct vicinity of the edges, that behavior first established by Nikuradse (reference 9) for the built-up turbulent flow in rectangular pipes. From this phenomenon, according to which at some positions particles farther removed from the wall have smaller velocities than those closer to the wall, Nikuradse derived the conclusion that in turbulent flow in pipes of noncircular cross section secondary flows are superimposed on the main flow. An explanation for the origin of such secondary flows at the edges of a cross section has been given by Prandtl (reference 8) which will be briefly reviewed here. Prandtl concludes from the above-mentioned facts that in the case of turbulent flow, there exists, in addition to the to-and-fro motion in the direction of maximum velocity drop, a still stronger to-and-fro motion at right angles to it, that is, in the direction of the isotachs. If this is the case, then it immediately follows that centrifugal forces thereby arise on the particles which move up and down along the curvatures of the isotachs. These forces always point toward one direction, namely the convex sides of the isotachs and are stronger the greater the curvature of the latter. The result is that the particles move outward along the angle bisectors of the corners of the cross section whereby the particles located at the corners are forced, for reasons of continuity, to deviate first laterally and then inwardly (see fig. 27). Since the particles flowing toward the corners possess a relatively large axial velocity, whereas conversely, the outwardly

flowing particles as a result of the friction at the wall
have a smaller axial velocity, this explains the unexpect-
ed appearance of the velocity profiles and hence the iso-
tachs. The question as to the reason for the turbulent to-
and-fro motion in the manner described is not answered by
this consideration of Prandtl as it leads to the more com-
prehensive question of the origin and inner mechanism
of turbulence in three dimensional flows and this in the
present stage of our turbulence investigation must still
remain unanswered.

     Returning to the consideration of the development of
the isotach curves in the entry run, it may be found in
cross section III (fig. 20) that the strong indentations
of the isotachs at the positions where the particles are
pushed inward have entirely disappeared. On the basis of
the Prandtl theory, this phenomenon may be explained by
the consideration that the particles with relatively small-
er axial velocities with increasing pipe length are pushed
aside into the interior of the flow and replaced by par-
ticles which have larger axial velocities. In cross sec-
tion IV (fig. 21) the secondary flow has taken in a greater
region at the edge so that at points whose distance is
about 20 mm from the wall the effect is already evident.
The particles with relatively smaller velocities pushed
toward the interior of the cross section apparently gradu-
ally give rise to the indentation of the isotachs that
heretofore were not affected by the secondary flow, so
that by the decrease in the radius of curvature the sec-
ondary flow may also be built up there. This process con-
tinues (see cross section V, fig. 22) until all instabil-
ities in the flow have disappeared. It is in cross section
0 (fig. 26) that the built-up turbulent velocity distribu-
tion is first found, there being superimposed over the en-
tire flow secondary flows that include a large region of
the cross section. As may be seen from the built-up iso-
tach curves at a section with shorter entry run, there is
no isotach picture that entirely coincides with that of
section 0, so that the correctness of the statement that
the profile of section 0 represents the fully built-up
turbulence still remains to be proven. This proof may be
given as follows: At the entrance to the initial run
there was built in a honeycomb similar to that in the circu-
lar initial run in the transition circle ⟶ rectangle.
In this manner the required initial run was shortened, so
that in section VII a velocity distribution could be es-
tablished which no longer varied with increasing length of
run and which agrees with that of cross section 0 measured
without honeycomb.

For a further characterization of the velocity devel-
opment along the entry run without honeycomb, there was
plotted the ratio of the mean velocity to the maximum in
the pipe axis as a function of the length of run. (See
fig. 28.) As shown by the figure, this ratio drops sharp-
ly from the value $c_{z_m}/c_{max} = 1.0$ at the beginning of the
run and this corresponds to equal velocity over the entire
cross section and then attains in cross section II and in
the following cross sections up to section VI, so long as
the secondary flows are restriced to the corners, an al-
most constant value. From section VI on, the secondary
flow gives rise to a change in the maximum velocity until
the latter maintains a constant value from section VIII on.
The first change in the ratio c_{z_m}/c_{max} is thus essen-
tially produced by the pure friction effects at the wall
whereas the subsequent lowering of this ratio may be as-
cribed to the effects of the turbulence of the flow.

In order to check whether the velocity distribution, also,
at least in the neighborhood of the walls of the individual
cross sections of the entry run, may be expressed by a sim-
ple exponential law of the form $c_z/c_{st} = a \eta^{1/n}$ (where η
is the shortest distance from the wall), there was plotted
by the method already described in section 6 (fig. 15), for
all the measured profiles in the measuring planes indicated
in figure 12, the logarithm of $10 c_z/c_{st}$ against the log-
arithm of the distance from the wall (fig. 29). In most
cases a straight line could be drawn through the points
thus obtained and the existence of a law of the above sim-
ple type established. The values of n for the different
profiles are given in table 3. Most of the values there
indicated are valid up to a distance from the wall of 15
millimeters. With increasing length of entry run, the ve-
locity distribution is well represented by these values
for an increasingly larger region of the cross-section
area. The dashes in the table in the case of some pro-
files indicate that for the latter no simple law could be
established. These are the positions at which the second-
ary flows gave rise to particularly extreme velocity rela-
tions at the wall. The development of the secondary flow
is especially responsible for the often irregular succes-
sion of the exponents in the various cross sections. In
the case of the fully built-up velocity profile in section
0, it was possible to establish throughout the value 1/7
for the exponent in agreement with the measurements of
Nikuradse (reference 9). The measure of the accuracy and
the region of validity of this 1/7 power law are indicated
in figure 29.

In concluding this section, there will also be given the drop in pressure head along the entry run, that is, in the z direction. For measuring this drop, three orifices are made at each section (b in fig. 8), one at the center of the bottom and two at the side walls of the rectangular pipe. The pressure readings of the three orifices gave very good agreement among themselves for each cross section. Pressure measurements at the interior of the cross sections were not taken, on account of the difficulty mentioned of applying the sphere apparatus. In figure 28 for an air velocity $c_{z_m} = 16.3$ m/s, the change in the pressure head h in m of gas column is plotted against the entry length. The pressure head for the length of run 0 is obtained by computation. Assuming for this position that the mean velocity $c_{z_m} = 16.3$ m/s extends over the entire cross section, there is found then, according to the Bernoulli law,

$$\frac{p}{\gamma_L} = -\frac{c_{z_m}^2}{2\,g} = -13.6 \text{ m gas column}$$

As may be seen from figure 28, the value so obtained fits in very well with the pressure head curve, the latter being almost straight. Only for the first half of the entry run is there found a greater steepness in the curve and hence a somewhat larger pressure loss. This additional loss is due mainly to the fact that the mean velocity head

increases from the value $\left(\dfrac{c_z^2}{2\,g}\right)_m = 13.6$ m gas column at

in the inital section to the value 15.4 meters air column at section VI. From section VI to section 0, there is no further rise. The mean velocity head is determined from the following relation

$$\left(\frac{c_z^2}{2\,g}\right)_m = \frac{1}{V}\int^F \frac{c_z^2}{2\,g}\,dv = \frac{1}{c_{z_m}F}\int^F \frac{c_z^3}{2\,g}\,df =$$

$$= \frac{4}{c_{z_m}F}\int\int \frac{c_z^3}{2\,g}\,dx\,dy$$

where V and F have the meaning already indicated and

x   is the distance from the midpoint of the velocity pro-
file, measured along the horizontal lines, and   y   is the
distance from the midpoint of the velocity profile meas-
ured along the vertical lines (fig. 17).   The evaluation
of this double integral was obtained graphically.   The
loss coefficient λ_{an} at the position of the entry run
at which the turbulent velocity profile was fully estab-
lished is obtained from the relation

$$\lambda_{an} = \frac{dh}{dz} \; \frac{1}{\frac{U}{4\,F} \; \frac{c_{zm}^2}{2\,g}}$$

and is found to have the value $\lambda_{an} = 0.0173$,   dh/dz   de-
noting the drop in pressure head in the   z   direction.

    A further and more detailed investigation of the ve-
locity and pressure relations in the entry run was dis-
pensed with as this would have carried the work beyond the
present limits.

VIII. THE MEASURED VELOCITY AND PRESSURE DISTRIBUTION

IN THE TRANSITION PIECES

    To present the test results in such a manner as to
bring out the development of the velocity distribution in
the individual cross sections of the transition pieces
is found to be not a very simple problem.   The profiles
measured in the same measuring plane at the different cross
sections have different base lengths as a result of the
deformation.   In order to be able to compare such profiles
with one another the velocity ratio $c_z/c_{z_{max}}$ is usually
plotted against the nondimensional value x/b,   where   x
is the distance from the profile midpoint and b   denotes
half the profile width.   By making this plot for the com-
parison profiles, it is then possible to say whether any
initial profile for the case investigated becomes flatter
or steeper.   By this method Nikuradse and Dönch (refer-
ences 1 and 2) plotted the profiles for convergent and di-
vergent rectangular channels and compared them, thereby
establishing the fact that velocity profiles of flows in
diverging channels always become steeper toward the center

whereas, in converging channels, they become flatter than in the case of parallel flow, a phenomenon which may be expected also in such cases from a consideration of the Bernoulli law.

The assumption underlying this procedure is that the expression $\int_0^b c_z \, dx$ for the profiles to be compared should have the same value, that is, that the discharge per second should be the same. This requirement is taken into account in the above-mentioned works of Nikuradse and Dönch insofar as their investigations were restricted to parts of the flow very close to the main axis of a rectangular section and so far removed from the boundary walls parallel to this axis that the flow built up there is practically plane and the quantity flowing through the region under consideration remains constant. This, however, is not true in the present case, since at all sections of the transition piece the flow is three-dimensional. For this reason, a direct comparison of the different profiles in a measuring plane is not possible. It is necessary rather to consider the velocity distribution over the entire cross section. The best indication of the velocity distribution and its variation by the cross-section deformation is therefore provided by the isotachs $(c_z/c_{max}$ or $c_z/c_{st} = $ constant) at the various cross sections, as was already found to be the case in the discussion of the velocity distribution and its development in the initial rectangular run. This method also leads to the possibility of characterizing the velocity distribution in the manner shown by the curves in figures 61 and 62, where the method of presentation employed for plane flows is applied to three-dimensional flows. These curves will be discussed more in detail later.

Tables 4 and 5 give as examples the manner of evaluating from the results of the sphere measurements the three velocity components c_x, c_y, and c_z, for the cross section 5 in the transitions $C \longrightarrow R(I)$ and $C \longrightarrow R(II)$. Figures 30 and 31 give the plots of the c_x and c_y components. These curves show nothing surprising and their shape is at least qualitatively as expected from the result of the change in shape at the walls, provided, of cource, no separation takes place. The particles flowing in the neighborhood of the principal axis are the least affected by the change in direction and therefore have the smallest

c_x and c_y components. The deflecting effect increases
gradually toward the wall and attains its maximum there.
Correspondingly, the c_x and c_y components increase with
decreasing distance from the wall and drop to zero when
very close to the wall since here the resulting velocity c
must also be zero.

Figures 32 and 33 show the plots of the c_z compo-
nents given in tables 4 and 5 for the various measuring
planes and on the same figures are also indicated for com-
pleteness the velocities measured with the pitot tube in
the neighborhood of the wall. From these it is possible
to obtain directly the isotach diagram for these cross
sections (figs. 36 and 43).

For all four transition pieces investigated, the maxi-
mum velocity at the axis of the transition piece remains
constant and the isotachs for the cross sections are so
drawn as to join points of equal value of c_z/c_{max}. For
normal flow, the value of c_{max} is 16.4 m/s for the tran-
sition circle ⟶ rectangle and 19.5 m/s for the transi-
tion rectangle ⟶ circle. It is superfluous to indicate
for all cross sections the velocity distribution in the
same detail as that for section 5 of the transition C ⟶
R(I) and C ⟶ R(II). As already remarked, the c_x and
c_y distributions show nothing unexpected so that the in-
dication of their values for each cross section may be dis-
pensed with. For the same reason, it is not necessary to
give the measured c_x and c_y components for the transi-
tion R ⟶ C(I) and R ⟶ C(II). Only the development
of the c_z distribution as a result of the transition is
of interest and will therefore be described. In figures
34 to 38, 41, and 48 to 55, are given the isotachs for the
two transition pieces and flow directions. The development
of the velocity distribution in the exit runs may be seen in
figures 39, 40, 46, and 47. With regard to the latter, it
is to be observed that the rectangular cross section III
is at a distance of 2.0 m from section 0 and section IV at
a distance of 4.2 m. The circular cross sections 13 and 15
indicated in figures 46 and 47 are respectively at the dis-
tances 2.5 and 4.5 m from cross section 10.

In the case of built-up turbulent velocity distribu-
tion in the entry run, three effects may be expected to
operate on the velocity distribution in the transition
pieces:

1. The locally arising retardations and accelerations.

2. The change in the turbulence mechanism.

3. The change in the ratio of the area of cross section to the friction boundary of the section.

As may be seen, particularly from the isotach diagrams for the transition piece II, the local retardations and accelerations have a large effect. In both transition pieces these effects arise in the sections 9 and 1. In the latter sections the flow which possesses only axial components c_z must either undergo transition to a flow which possesses also c_x and c_y components or a flow with all three components must be converted into a purely axial one, depending on the direction of transition. To obtain a change in velocity pressure, differences are required as is known across the velocity direction. To understand the development of the isotach curves, it is therefore necessary to have the accurate pressure distribution at each cross section of the transition pieces. This distribution, as previously mentioned, was also measured with the sphere apparatus and is given on figures 56 to 60 for the transitions $C \longrightarrow R(I)$ and $C \longrightarrow R(II)$. The pressure distributions for the transitions $R \longrightarrow C(I)$ and $R \longrightarrow C(II)$ have fundamentally the same appearance, so that they need not be given. On these figures are shown the isobars, the numbers at each curve giving the reduced pressure heads in meters air column for a flow of air with mean velocity of $c_{z_m} = 14.0$ m/s. In the case of transition piece I, different pressures at different points of the same section are found only for the sections 9 and 1 (figs. 56 and 57). For all other intermediate sections of transition piece I, no differences in pressure were found over the same section. From the discussion given below of the development of the isotach diagrams in transition piece I, it will be seen, however, that in the cross sections immediately preceding and following sections 9 and 1 near the walls there must be a small pressure rise or drop as an effect of the deflection of the flow. The aftereffects are stronger in the more steeply built transition piece II (fig. 59). As a result of the stronger deflections in the case of transition piece II, higher pressure differences arise over the cross section situated at the deflecting position, which still show up so strongly at section 5 that no unique constant pressure could be measured over the entire cross section. Cross sections 0 and 10 of transition piece II are again so far removed from the deflecting position that for these a distribution with con-

stant pressure could be obtained for both flow directions.
Also, there should be observed, with regard to the pressure
distributions, that they have the typical appearance to be
expected. At each cross section at which there is a deflec-
tion, four pressure regions are built up along the wall,
namely, two low and two high pressure regions as compared
with the pressure at the center portions of the cross sec-
tions. These high and low pressures are of different mag-
nitude, depending on the measuring plane in which they are
considered, since deflections of varying magnitude occur
in the different measuring planes. Thus, for example, in
section 9 of the transition C ⟶ R(I) and C ⟶ R(II), the
particles lying in the measuring planes 5, 6, and 7, must
be more deviated than those in measuring planes 4 and 8.
For this reason, the high pressures arising at the wall in
measuring planes 5, 6, and 7, are greater than in 4 and 8.
The same considerations can also be applied for the nega-
tive pressure arising at the side walls. In the interior
of the cross section there are superimposed, according to
the position of the point considered, the effects of the
positive and negative pressures, so that a resultant pres-
sure distribution is built up there.

IX. MORE DETAILED CONSIDERATION OF THE VELOCITY DISTRIBUTION

Figures 35 to 40, 42 to 47, 49, and 51, show first of
all that the isotachs adjust themselves very rapidly to the
new cross section contours. Particularly do the isotachs
in the immediate neighborhood of the walls follow the con-
tours of the cross sections closely, even at the corners,
whereas those more at the center are naturally not so strong-
ly affected. An exception is formed in the case of the iso-
tachs in cross sections 1 and 9 for the transitions C ⟶
R(II) and R ⟶ C(II) (figs. 48, 50, 52, and 54). In
the case of the latter cross sections the large pressure
differences due to the strong flow deflections result in a
completely different behavior. Thus, for example, in the
transition C ⟶ R(II), the air in measuring planes 12,
13, or 14, flowing into section 9, meets a region with
greatly lowered pressure and therefore the velocity head
increases. Relatively high velocities arise therefore at
the wall also, so that a very rapid velocity rise takes
place from the wall to the interior of the section. The
reverse is true for the air in the measuring planes 4, 5,
6, 7, etc., where, due to the deflection, there arises
at section 9 a high pressure as compared with that at the
center portions so that the air particles must give up
kinetic energy in order to be pushed into the region of

higher pressure. This explains at least qualitatively the very slow rise of the velocity as shown by the isotachs. This phenomenon, according to which the velocity distribution in the case of accelerated flow becomes flatter and, in the case of retarded flow, sharper, is in full agreement with the test results already mentioned of Nikuradse and Dönch for plane flow.

Similarly, the irregular rise of the velocities at the wall is to be ascribed to local retardations or accelerations, as was found for cross sections 7, 5, and 3, of the transition C ⟶ R(I) (figs. 35, 36, and 37) and in cross section 5 of the transition C ⟶ R(II), depending upon whether the velocity distribution considered is in a horizontal or perpendicular measuring plane. As already remarked, the pressure does not immediately equalize after the deflection, so that also in the cross sections following the deflection cross section, accelerations and retardations occur. However, in cross section 5 of the transition C ⟶ R(I), there already hardly appear pressure differences over the cross section. The isotach diagram nevertheless shows also for this cross section that at the side walls the velocity drop is smaller than at the bottom wall while it would be expected that the conversion of the kinetic energy into pressure energy and conversely ends there so that at all positions of the cross section there is the same velocity rise at the wall. This apparent contradiction finds its explanation in the consideration of the losses occurring during the energy conversions. It is generally known that a retardation of a medium along a friction wall leads to large pressure losses while acceleration leads to only small losses. Now such retardations along the wall occur in the measuring planes 12, 13, 14, etc., in the cross sections immediately following section 9 so that the flowing particles there lose velocity not only as a result of the conversion of kinetic to pressure energy but also through the friction of the wall and there thus arises an additional velocity loss. These effects may naturally still be observed in section 5. They can only disappear if new energy can be transported from the "healthy" flow in the interior of the sections to the particles at the wall. This transport of kinetic energy can only take place through the turbulent exchange. This is confirmed by consideration of the isotach diagrams for the transition R ⟶ C(I). It is found that, for example, in cross section 5 there are no effects in the velocity distribution as a result of retardations along the wall but that the velocity rise in the neighborhood of the wall is the

same along the entire circumference although also for this
transition retardations occur along the wall in the cross
sections ahead of cross section 5. This different behavior
is explained by the fact that through the existence of the
secondary flow the turbulence is so strong that the slowed-
up particles at the wall can always be supplied with new
kinetic energy. That the fluctuating turbulent velocities
in the transition rectangle ⟶ circle are greater than for
the reverse transition must also be concluded from the much
stronger fluctuations to which the fluid columns of the micro
manometers used for measuring the velocity were subjected.

From the isotach curves for the transitions R ⟶ C(I)
and R ⟶ C(II) it may also be clearly made out that the
secondary flows are gradually destroyed on receding from
the walls and a new turbulence mechanism comes into play.
The effects of the secondary flows on the isotachs do not
vanish completely, however, until after the flow has passed
through a considerable length of pipe. Thus it may be seen
that even after a circular exit run of 4.5 m the isotachs
in the center portions of cross section 15 still show the
characteristic indentations of the secondary flows. From
this it may be concluded that the velocity distributions in
the intermediate cross sections of a transition piece for
regions not directly close to the wall depend essentially on
the velocity distribution at the entrance. Since the veloc-
ity distribution at the end cross section is produced by the
turbulence mechanism acting there, the result may also be
expressed by saying that the change in the entire turbulence
mechanism associated with the change in the cross-section
shape hardly affects the velocity distribution in the tran-
sition piece itself. This is also understandable for the
case of our relatively short transition lengths since it is
known that the turbulent mechanism for example in the flow
through a circular pipe must act on an initial length of
run of about 90 diameters in order to produce a built-up
velocity distribution.

In the present state of turbulence investigation for
three dimensional flows it is unfortunately not yet possi-
ble to give more accurate numerical data on the changed
turbulence mechanism produced by the transition piece which
data would consist of the accurate mixing length distribu-
tion and its variation for each cross section. For this
computation it would be necessary to extend the shear stress

law $\tau = \rho \, l^2 \left(\dfrac{dc_z}{d\eta} \right)^2$ mentioned in section VI for the two-

dimensional case to three-dimensional flows (references

7, p. 325, and 8). The law thus extended has not yet been tested even for fully built-up three-dimensional turbulent flow as would be necessary before application is made to varying velocity distributions such as those in the transition pieces. Even if it were succeeded in computing the mixing length distribution according to the extended shear stress law the true turbulence mechanism would not yet be described since through the occurrence or vanishing of secondary flows further forces arising from the oscillation of the particles in the direction of the isotachs would be effective by acting on the flow which forces are not taken into account by the shear stress law and for which quantitative information is still impossible to obtain. It is therefore evident why the attempt to give a numerical account of the turbulence mechanism and its variation due to the transition will be dispensed with here.

For a further characterization there was therefore employed the method already applied for two-dimensional flow, namely, that of determining the exponents for the velocity distribution (c_z components) in the neighborhood of the wall of the various sections for the various measuring planes. These exponents were found in the same manner as for the rectangular entry runs. The values of the denominator n are collected in tables 6 and 7 for the two transition pieces investigated and for each direction. As may be seen, for all profiles whose measuring planes were sufficiently far removed from the parallel walls so that there was a unique distance from the wall an exponent could be determined at least for the layers next to the wall. No separations from the wall were found for any transition. From table 6 for the transition $C \longrightarrow R(I)$ it may be seen that wherever local retardations occur the exponent $1/n$ is greater than at the initial profile whereas for local accelerations it remains the same or becomes somewhat smaller. This fact is even more evident from a consideration of table 6 for the transition $C \longrightarrow R(II)$ where for accelerations values such as $1/21.0$ occur for the exponents and for retardations values such as $1/2.6$ while in the initial profile the value $1/8.3$ holds over the entire profile. This phenomenon is to some extent in contradiction with the measurements of Nikuradse. The latter finds in his tests on convergent and divergent channels of small opening angle that the $1/7$ power law obtaining at the initial section is not changed by retardation or acceleration. Besides the fact that the results of the two works are not directly comparable in this re-

spect since in Nikuradse's case there was retardation or
acceleration over the entire cross section whereas in our
case only local retardations or accelerations with constant
pressure occur over a cross section this difference may
perhaps be explained by table 7 in which the exponents are
given for the transition R——→ C(I). There it will be
found that the exponents in the accelerated or retarded
portions at least for the portions of the wall lying in
the neighborhood of the major and minor axes of the cross
section are not different but equal to that at the initial
profile. This different behavior as compared with the
transition C——→ R(I) is again to be ascribed to the
secondary flows. Since Nikuradse, as already remarked, in-
vestigated only the velocity distributions in a middle
plane of the rectangular channel and the angles of diver-
gence were of the same order of magnitude as those of tran-
sition piece I it is possible that the secondary flows oc-
curring in his case and hence the increased turbulence
similarly led to the result that for all cross sections in
the middle plane of the convergent or divergent channel
the 1/7 power exponential law was found to hold.

      In concluding this section there will be considered
the effect of the change in magnitude of the friction areas
due to the change in shape of the cross section on the ve-
locity distribution. Figures 61 and 62 show curves which
were obtained from the isotach diagrams the values of
c_z/c_{max} being plotted against the ratio expressed in per-
cent of the area enclosed by an isotach to the entire cross
section area, the areas being obtained by planimeter. Com-
parison of two such curves at different cross sections is
sufficient to show in general whether the velocity distri-
bution over the entire cross section becomes fuller or more
tapered as a result of the deformation. For the transitions
C ——→ R(I) and C ——→ R(II) it may be seen from figure 61
that the velocity distribution becomes as was expected more
tapered in cross section 0. Similarly the flattening of
the velocity distribution through the transition R——→ C(II)
(fig. 62) quite corresponds to the expectation. On the
other hand it is surprising that for the transition R——→
C(I) the velocity distribution becomes more tapered, since
in this case it would also be assumed that as a result of
the decrease in the frictional circumference the proportion
of particles with relatively small velocities would become
smaller and there would thus occur a flattening of the ve-
locity distribution. If account is taken of the fact, how-
ever, that the changed velocity distribution due to the

cross section deformation depends not only on the change
in magnitude of the friction areas but also on the previ-
ously mentioned two effects, namely, the local retardations
and accelerations and the turbulence mechanism, which play
an important part then the curves of figures 61 and 62 may
be explained in the following manner: As already said,
for all four transitions in the cross sections 0 and 10
no pressure differences were found in the cross section
plane. There is thus available for each air particle the
same amount of pressure energy for its path through the
corresponding transition piece. The particles with small
velocity near the wall lose still more kinetic energy while
those more toward the center with relatively larger veloc-
ities are entirely unaffected or only slightly affected by
these local retardations and so flow with considerably
smaller pressure loss so that they are even accelerated
somewhat by the excess pressure energy. If in addition
there is an increase in the frictional perimeter of the
cross section during the transition the two effects are
superposed in the same sense so that for such cases the
plotted velocity distribution becomes more tapered (fig.
61). For the converse case, that is, where the friction
surface is reduced as a result of the cross section defor-
mation no such general statement can be made for the ve-
locity distribution since it is a question as to which of
these two effects is preponderant. From figure 62 it must
therefore be concluded that for the transition R———► C(I)
the effect of the arising local retardations is preponder-
ant over that of the decrease in the perimeter of the
cross section while for the transition R———► C(II) the
reverse is true. The reason for this different behavior
may perhaps be sought in the fact that the secondary flow
due to the short transition length of piece II is still
so strong even after the transition that the particles
stopped at the wall can immediately receive new kinetic
energy by the turbulence interchange while as a result of
the larger transition length of piece I the secondary
flows are gradually destroyed already at the walls of the
transition piece so that this turbulence interchange cannot
take place to the same extent as for the transition R———►
C(II). Considering figures 61 and 62 in the light of this
view it may be said that only for transition R———► C(II)
is there any directly evident effect of the magnitude of
the cross section friction perimeter.

X. THE PRESSURE DROP IN THE TRANSITION PIECES

On figures 63 and 64 there is plotted for all four investigated cases the drop in pressure at the wall in meters air column against the pipe length for a portion of the entry run, the transition piece and the exit run. The difference in the initial pressure of cross section 10 for the transitions $C \longrightarrow R(I)$ and $C \longrightarrow R(II)$ is to be ascribed to the fact that in the transition $C \longrightarrow R(II)$ the entry run was somewhat shorter than for transition $C \longrightarrow R(I)$. The measured wall pressures for the sections 9 and 1 of the transition pieces which pressures are different depending on whether the side or bottom orifices are used for the measurement are omitted. The variation of the wall pressure along an axial section of the transition piece naturally provides no measure for the losses occurring if the pressure in the interior of the cross section is not constant and does not agree with that at the wall. The typical appearance of these curves is for all cases the same. As a result of the conversions occurring in the transition pieces of kinetic into pressure energy and conversely the pressure in the first portion of the transition piece rises more strongly than in the entry run and again sinks toward the end of the transition. These energy conversions come to an end only when the pressure in the exit run is again linear. For all transitions this condition obtains after a final length of run of $l_a' = 0.5$ m. Considering the pressure difference between the inlet and outlet flange of the transition piece without taking account of the partial pressure gain in the exit run as pressure head loss h_v of the transition piece of total length l_v there is obtained by substituting the mean hydraulic diameter of

$$ d_{h_v} = 4 \ \frac{F}{U} = 4 \ \frac{F_C + F_R}{U_C + U_R} = 0.173 \text{ m} $$

into the loss equation

$$ h_v = \lambda_v \ \frac{l_v}{d_{h_v}} \ \frac{c_{z\,m}^2}{2 \ g} $$

the values given in table 8 for the loss coefficient.

There are also given in this table values for a λ_a' that was determined for the 0.5 m piece l_a' of the exit run in which further energy conversions take place. On comparing the values of λ_v and λ_a' it will be seen that these values come out considerably smaller for the transitions $R \longrightarrow C(I)$ and $R \longrightarrow C(II)$ than for the transitions $C \longrightarrow R(I)$ and $C \longrightarrow R(II)$. The reason for this difference must be the increase in the mean velocity head $c_{z_m}^2/2g$ in the transitions $C \longrightarrow R(I)$ and $C \longrightarrow R(II)$ which in the case of the transitions $R \longrightarrow C(I)$ and $R \longrightarrow C(II)$ remains practically constant. The mean velocity head in the different cross sections was deter-mined by the method already mentioned in the considera-tion of the pressure variation along the rectangular entry run. For the transitions $C \longrightarrow R(I)$ and $C \longrightarrow R(II)$ there was obtained an increment in the mean velocity head from 10.3 m in cross section 10 to 11.45 m in cross sec-tion 0.

If it is desired to find which of the two transition pieces is better with regard to the pressure losses we should not restrict ourselves naturally to the comparison of the values of λ_v and λ_a' but must compare the sums formed of the products $\lambda_v \dfrac{l_v}{d_{h_v}} \dfrac{c_{z_m}^2}{2\,g}$ and $\lambda_a' \dfrac{l_a'}{d_{h_a}} \dfrac{c_{z_m}^2}{2\,g}$.

If this comparison is carried through for the transitions $C \longrightarrow R(I)$ and $C \longrightarrow R(II)$ it will be found that the transition piece II causes a smaller pressure loss and so is better suited for the transition circle $\longrightarrow$ rectangle. For the direction of transition rectangle $\longrightarrow$ circle the result is arrived at that both transition pieces are equivalent. This at first rather surprising result be-comes explainable on considering figure 55 from which it may be seen that for cross section 10 in the transi-tion $R \longrightarrow C(II)$ the still very strong secondary flow in the immediate neighborhood of the wall in section 10 of transition $R \longrightarrow C(II)$ leads to very large velocity drops which naturally result in increased losses and bring about the result that the transition piece II in spite of its smaller length produces the same pressure drop as transi-tion piece I.

It must still be pointed out that these findings with regard to the relative efficiency of the transition pieces are justified only in the case where there exists a built-up turbulent velocity distribution at the entrance to the

transition piece. If other velocity distributions exist
the question of the relative efficiency of the transition
pieces may be differently decided since different velocity
distributions result in different energy distributions and
the latter are of very great effect on the pressure loss.

In conclusion attention is called to the loss coeffi-
cients λ_{an} and λ_a in table 8. These coefficients cor-
respond to those parts of the entry and exit runs in which
there is a linear relation between pressure loss and pipe
length. The two columns agree very well and this moreover
brings out the justification for the customary introduc-
tion of the hydraulic diameter for the determination of the
pressure loss of pipes of noncircular cross section.

XI. SUMMARY

In this paper the results are presented of pressure
and velocity distribution measurements in transition
pieces in order to investigate the effect of the change
in shape of the cross section contour for equal area of
cross section. For the two investigated transition pieces
which differed in their transition lengths a circular
cross section is deformed into one of rectangular perim-
eter. Since in each transition piece measurements were
taken for flows in each direction four different cases
were considered. The entry runs connected ahead of the
transition pieces were chosen of such length as to obtain
a built-up turbulent velocity distribution at the end of
the run. For this purpose velocity measurements in the
entry run of the rectangular pipe were necessary. For
circular pipe section these velocity distributions are
sufficiently well known.

The velocity distributions measured in the various de-
formed cross sections are brought out by means of isotach
diagrams. The isotachs at each cross section very rapidly
assume the shape of the corresponding cross section provid-
ed the local retardations and accelerations do not become
too large. There is shown the existence of an exponential
law for the velocity in the neighborhood of the wall in
each of the measuring planes parallel to the axis.

The most important factors affecting the velocity and
pressure distribution in the transition pieces are named

and it is attempted to show their effects at least quali-
tatively. The greatest effect is that produced by the
local retardations and accelerations. It is particularly
difficult to determine the effect of the change in the
turbulence mechanism through the cross section deformation
on the velocity distribution. Since in a rectangular pipe
the turbulence of the flow shows up directly by the pres-
ence of secondary flows it is concluded from the measure-
ments for the transition rectangle ⟶ circle. that the
change in the turbulence mechanism due to the cross sec-
tion deformation does not very greatly affect the velocity
distribution in the transition cross sections themselves.
A third effect indicated is the change in the friction
perimeter of the cross section due to the deformation.
The establishment of this effect is also rendered diffi-
cult by the fact that there is no method for finding this
effect independently of the other two. An important fact
established is that in no transition piece did separation
from the wall occur.

At the conclusion of the paper are given the loss co-
efficients for the two transition pieces in both flow di-
rections.

Translation by S. Reiss,
National Advisory Committee
for Aeronautics.

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Table I
Cross section areas for piece I

No. of section	Area cm²
10	288,6
9	288,6
8	284,6
7	283,6
6	282,2
5	282,5
4	281,5
3	281,4
2	281,0
1	285,5
0	284,5

Table III
Value of n for various positions of entry run

Measuring plane	Cross section								
	I	II	III	IV	V	VI	VII	VIII	0
12	8,1	6,0	6,0	8,4	6,0	8,5	11,0	8,3	7,0
13	9,0	5,2	6,5	7,4	6,0	8,3	7,5	8,3	7,0
14	6,5	6,0	6,5	5,5	—	7,3	7,4	7,2	7,0
14a	7,0	6,5	6,5	6,5	—	7,3	8,0	8,3	7,0
14b	5,0	6,0	6,8	6,3	—	7,0	9,0	8,0	7,0
14c	5,0	5,2	7,0	6,6	—	6,5	10,4	8,0	7,0
6	8,0	5,5	12,5	7,2	8,4	13,5	9,0	8,3	7,0
7	8,0	6,0	12,5	6,5	9,5	11,1	9,3	8,5	7,0
8	8,0	6,2	9,7	6,7	8,4	10,1	9,4	7,9	7,0
9	14,0	6,0	8,7	6,7	11,0	9,3	8,3	7,8	7,0
10	8,5	6,6	9,2	6,7	11,0	8,2	8,0	7,0	7,0
11	6,5	—	8,8	6,3	—	8,4	6,3	6,6	7,0
11a	7,0	—	8,5	6,7	—	7,6	5,9	5,3	7,0
11b	8,0	—	8,3	7,7	13,6	6,5	5,4	5,6	7,0

Table II
Cross section areas for piece II

No. of section	Area cm²
10	288,6
9	288,6
5	281,0
1	282,4
0	287,6

Table VIII
The loss coefficient λ.

Transition	λ_{an} Entry run	λ_v Transition	λ_a' Exit run	λ_a Exit run
C→R (I)	0,0162	0,0248	0,0266	0,01665
C→R (II)	0,0162	0,0237	0,0228	0,01665
R→C (I)	0,0173	0,0106	0,0117	0,0171
R→C (II)	0,0173	0,0199	0,00934	0,0171

Table IV
Evaluation of sphere measurements at section 5 of transition C $\longrightarrow$ R(I)

Measuring point		α Deg.	δ Deg.	ψ Deg.	c m/s	c_x m/s	c_y m/s	c_z m/s	p/γ_L m GS
y	x								
12	6	0	0	0	16,38	0	0	16,38	— 41,6
12	7	+ 0,8	0	+ 11,7	16,38	+ 0,223	— 0,046	16,37	— 41,5
12	8	+ 1,5	— 1	+ 22,2	16,4	+ 0,50	+ 0,101	16,20	— 41,6
12	9	+ 2	— 2,5	+ 31,6	15,92	+ 0,835	+ 0,298	15,38	— 41,4
12	10	+ 2,5	— 4	+ 39,6	15,12	+ 1,185	+ 0,331	15,08	— 41,5
12	11	+ 4,5	— 5,1	+ 44,8	12,05	+ 1,395	+ 0,055	11,95	— 41,6
13	6	+ 0,5	+ 0,5	+ 0,1	16,35	+ 0,142	+ 0,143	16,32	— 41,7
13	7	+ 2,5	+ 1,1	+ 9	16,35	+ 0,653	— 0,44	16,32	— 41,6
13	8	+ 4	+ 0,6	+ 18,1	16,32	+ 1,02	— 0,533	16,30	— 41,4
13	9	+ 5	0	+ 26,3	16,25	+ 1,265	— 0,627	16,20	— 41,6
13	10	+ 5,5	— 1	+ 33,5	15,35	+ 1,371	— 0,577	15,27	— 41,6
13	11	+ 4,5	+ 4,5	+ 39,5	11,94	+ 1,325	— 0,125	11,88	— 41,7
14	6	+ 0,5	+ 1,2	+ 0,7	16,29	+ 0,137	— 0,375	16,21	— 41,5
14	7	+ 3	+ 2	+ 7,5	16,15	+ 0,763	— 0,658	16,12	— 41,6
14	8	+ 4,5	+ 1	+ 15,2	15,98	+ 1,132	— 0,598	15,9	— 41,7
14	9	+ 6	0	+ 22,5	15,62	+ 1,42	— 0,588	15,55	— 41,6
14	10	+ 6,5	+ 1,6	+ 29,3	14,76	+ 1,665	— 0,443	14,65	— 41,7
15	6	+ 1,5	+ 1	+ 1,3	14,78	+ 0,398	— 0,247	14,72	— 41,5
15	7	+ 4	+ 2	+ 6,4	14,72	+ 0,96	— 0,624	14,7	— 41,5
15	8	+ 5,5	+ 1,5	+ 13,4	14,6	+ 1,277	— 0,696	14,5	— 41,6
15	9	+ 8	+ 0,5	+ 20	13,4	+ 1,71	— 0,491	12,8	— 41,6

* The measuring points are the intersections of the three planes.
1, Cross section 5.   2, Horizontal (x) plane.   3, Vertical (y) plane.

Table V
Evaluation of sphere measurements at section 5 of transition C $\longrightarrow$ R(II)

Measuring point		α Deg.	δ Deg.	ψ Deg.	c m/s	c_x m/s	c_y m/s	c_z m/s	p/γ_L m GS
y	x								
12	6	0	0	0	16,38	0	0	16,38	— 36,6
12	7	+ 2	— 0,25	+ 10,8	16,38	+ 0,570	— 0,053	16,34	— 37,0
12	8	+ 4	— 1,5	+ 20,8	16,28	+ 1,205	— 0,0018	16,20	— 37,4
12	9	+ 6	— 3,3	+ 29,8	15,85	+ 1,88	— 0,0039	15,7	— 37,75
12	10	+ 6,2	— 6	+ 37,5	14,85	+ 2,2	+ 0,274	14,65	— 37,7
13	6	0	+ 2,5	0	16,38	0	— 0,712	16,38	— 37,2
13	7	+ 2,5	+ 2,1	+ 9,0	16,37	+ 0,606	— 0,72	16,32	— 37,0
13	8	+ 5	+ 1,9	+ 17,8	16,1	+ 1,175	— 0,918	16,02	— 37,4
13	9	+ 7	+ 1,7	+ 25,6	15,78	+ 1,90	— 0,416	15,62	— 38,0
13	10	+ 8,5	+ 5,3	+ 32,9	14,68	+ 2,54	— 0,0168	14,38	— 37,8
14	6	+ 0,7	+ 6	+ 0,2	15,85	+ 0,178	— 1,66	15,72	— 37,3
14	7	+ 4,5	+ 5,5	+ 8,2	15,8	+ 1,00	— 1,665	15,63	— 37,3
14	8	+ 8	+ 4	+ 15,8	15,6	+ 1,785	— 1,635	15,4	— 37,7
14	9	+ 11	— 0,2	+ 22,9	15,12	+ 2,667	— 1,077	14,82	— 38,0
14	10	+ 13,5	— 5,7	+ 29,1	14,4	+ 3,62	— 0,387	13,92	— 38,2
14	10b	+ 13,5	— 8,5	+ 32,1	13,45	+ 3,68	— 0,022	12,95	— 38,3
15	6	+ 1	+ 11,25	+ 0,6	14,77	+ 0,228	+ 2,28	14,45	— 37,5
15	7	+ 6,2	+ 11,2	+ 7,4	14,4	+ 1,122	— 2,293	13,90	— 37,8
15	7	+ 13	+ 10	+ 14	13,58	+ 2,25	— 3,0	13,0	— 38,2

* The measuring points are the intersections of the three planes
1, Cross section 5.. 2, Horizontal (x) plane. 3, Vertical (y) plane.

Table VI
Value of n at different sections of transitions C → R(I) and C → R(II)

Measuring planes	Transition C→R (I) Cross section									Transition C→R (II) Cross section				
	10	9	7	5	3	1	0	III	IV	10	9	5	1	0
12	8,3	9,5	5,8	5,8	6,0	6,4	8,0	6,0	7,0	8,3	21	8,0	3,5	8,0
13	8,3	9,0	6,5	6,0	6,0	6,5	8,0	6,0	7,0	8,3	17	8,5	4,5	8,0
14	8,3	8,5	7,5	6,0	7,6	6,8	8,0	7,0	7,0	8,3	16	9,0	5,3	8,0
14b	8,3	8,0	8,0	8,0	8,0	8,0	8,0	8,0	7,0	8,3	14	10,0	8,8	4,5
6	8,3	7,2	8,0	8,0	8,0	8,5	8,0	7,0	6,8	8,3	2,6	7,4	21,5	8,0
7	8,3	7,5	8,0	8,0	8,0	8,5	8,0	7,0	6,4	8,3	3,0	8,0	18	8,0
8	8,3	7,5	8,0	8,0	8,0	8,0	8,0	7,0	6,5	8,3	6,5	8,0	13	8,0
9	8,3	7,7	8,0	8,0	8,0	7,6	8,0	7,3	7,0	8,3	8,0	8,0	7,6	8,0
9b	8,3	8,0	8,0	8,0	8,0	7,4	8,0	7,3	7,0	8,3	9,5	4,5	8,0	7,5
10	8,3	8,0	8,0	8,0	8,0	7,0	8,0	7,0	+	8,3	13,0	4,0	8,0	7,0
10b	—	—	—	8,0	8,5	6,8	8,0	7,3	+	—	—	3,9	6,5	6,0
11	—	—	—	—	11,0	6,7	9,0	7,5	+	—	—		4,0	5,6
11a	—	—	—	—	—	6,5	9,2	7,5	+	—	—		4,0	5,5

+ Signifies that for these measuring planes no exponential low could be established since the secondary flows give rise to extreme velocity distributions near the wall.

− Signifies that the corresponding measuring plane lies outside the cross section.

Table VII
Value of n at different sections of transitions R → C(I) and R → C(II)

Measuring planes	Transition R→C (I) Cross section								Transition R→C (II) Cross section				
	0	1	3	5	7	9	10	15	0	1	5	9	10
12	7,0	7,0	7,0	7,0	7,0	7,0	6,4	6,4	7,0	5,0	16,0	20,0	7,0
13	7,0	7,0	7,0	7,0	7,0	7,0	6,2	7,1	7,0	5,5	16,0	20,0	12,0
14	7,0	6,7	7,2	7,1	7,2	7,5	6,0	6,9	7,0	4,5	17,0	20,0	12,0
14b	7,0	6,8	7,2	7,0	7,2	7,2	6,3	6,3	7,0	5,0	+	14,0	12,0
6	7,0	7,0	7,0	7,0	7,0	7,0	7,0	6,8	7,0	21	5,5	5,7	6,7
7	7,0	7,0	7,0	7,0	7,0	7,0	6,7	6,8	7,0	20	6,8	5,7	7,5
8	7,0	7,3	7,0	7,0	6,7	6,5	6,5	7,3	7,0	13,4	7,0	5,6	7,0
9	7,0	7,2	7,2	7,0	6,3	6,2	6,2	7,0	7,0	10,0	6,8	5,2	7,3
9b	7,0	7,0	7,2	7,0	6,9	6,3	6,0	6,7	7,0	9,0	3,8	4,7	9,7
10	7,0	7,0	7,2	6,9	7,8	7,0	6,5	7,4	7,0	7,5	3,5	12,0	8,3
10b	7,0	7,0	7,6	7,0	—	—	—	—	7,0	7,5	2,0	—	—
11	7,0	7,0	6,8	—	—	—	—	—	7,0	7,3	2,0	—	—
11a	7,0	7,0	—	—	—	—	—	—	7,0	7,0	—	—	—

+ Signifies that for these measuring planes no exponential low could be established since the secondary flows give rise to extreme velocity distributions near the wall.

− Signifies that the corresponding measuring plane lies outside the cross section.

From free atmosphere   To suction pipe of blower
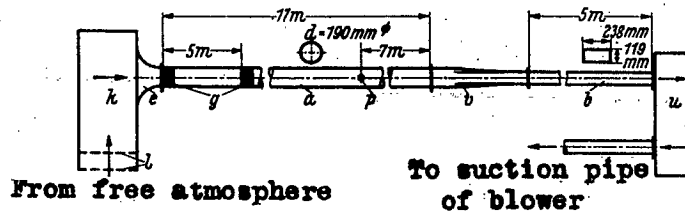
a, Entry run of circular section
b, Exit run of rectangular cross section
d, Diameter of circular pipe
e, Inlet throat
g, Honeycomb

k, Suction chamber
l, Perforated sheet
p, Prandtl tube
u, Deflecting chamber
v, Transition piece

Figure 1.- Test set-up for the deformation of a circular into a rectangular cross section.
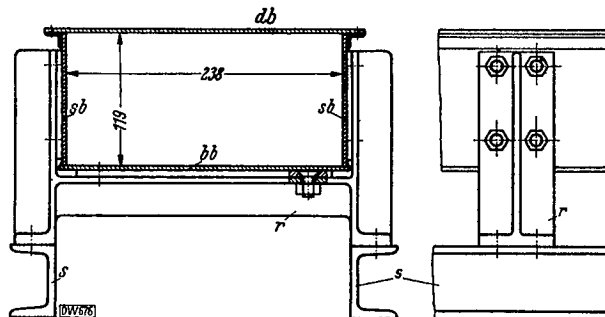
r, Frame
s, U-beam

bb, Bottom sheet
sb, Side sheets

db, Cover plate

Figure 2.- Cross section through rectangular channel.

From free atmosphere   To suction pipe of blower
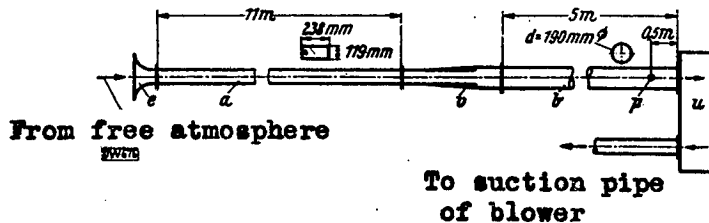
a, Entry run of rectangular cross section
b, Exit run of circular cross section
d, Diameter of circular pipe

e, Inlet throat
p, Prandtl tube
u, Deflection chamber
v, Transition piece

Figure 3.- Test set-up for the deformation of the rectangular into a circular cross section.
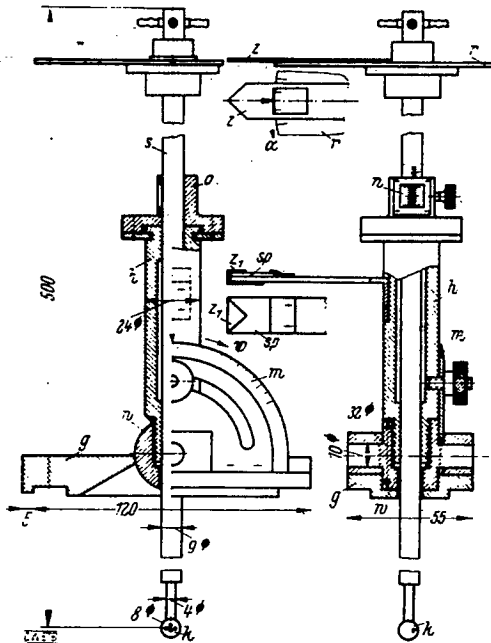
g, Base plate
h, Support
k, Sphere
m, Graduated circle for ψ
n, Vernier
o, Head piece
r, Graduated disk
s, Shaft of sphere
w, Cylinder
z, z_1 indicators
sp, Mirror

Figure 4.- Pressure sphere
apparatus.
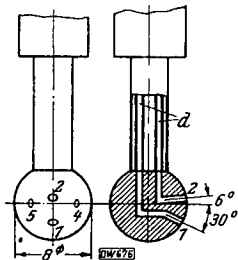
d, Pressure tubes;
1 to 5 orifices
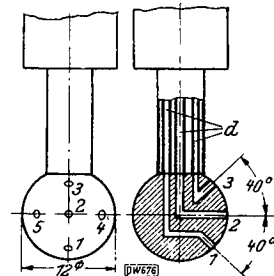
Figure 5a.- Pressure sphere employed
of 8 mm diameter with
four orifices.

Figure 5b.- Pressure sphere of 12
mm diameter with five
orifices as previously employed.

Figure 7.- Pitot tube.
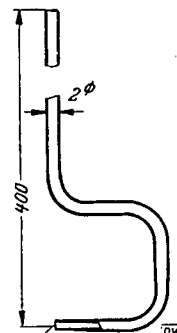
Inside diameter = 0.5 mm
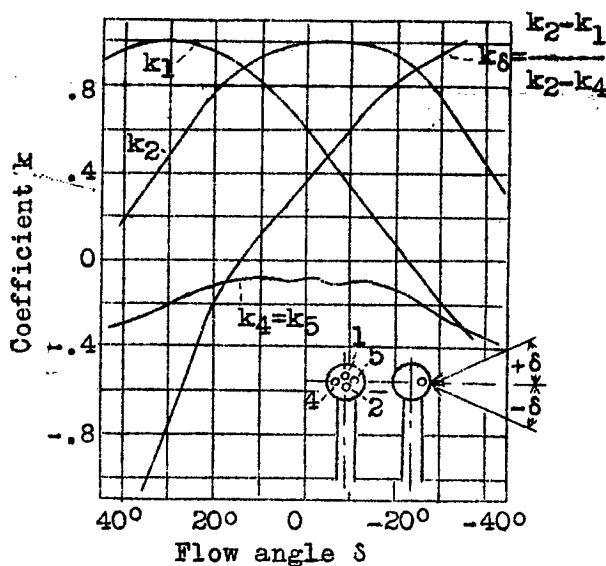
Figure 6.- Calibration curve
            of the pressure
sphere employed with four
orifices (see fig. 5a).

Figure 13.- Initial velocity
              profile in
circular cross section 10
in the transitions $C \rightarrow R(I)$
and $C \rightarrow R(II)$.

Figure 14.- Mixing length
              distribution as
a function of the distance
from the wall for cross
section 10 of the transitions
$C \rightarrow R(I)$ and $C \rightarrow R(II)$.
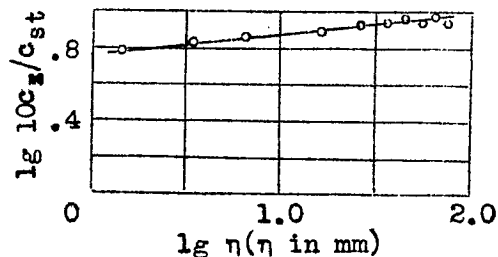
Figure 15.- Determination
              of the exponent
$1/n$ for the velocity
distribution in cross
section 10 of the tran-
sitions $C \rightarrow R(I)$ and
$C \rightarrow R(II)$.

Cross section ⑩

Contours of cross
sections ① to ⑩
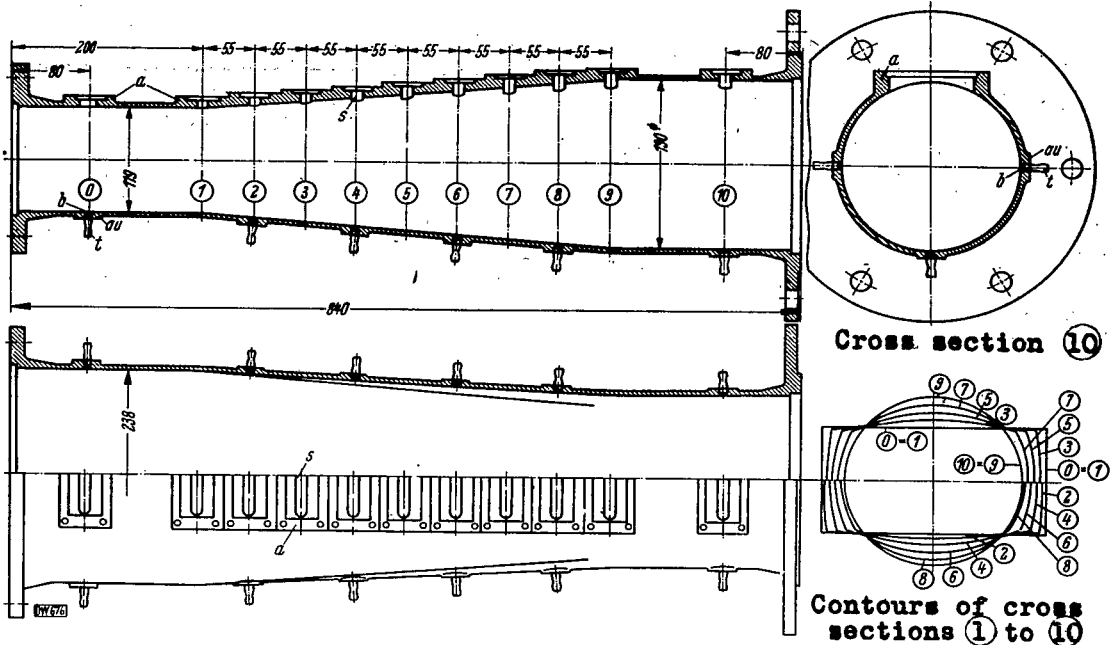
a, Seats for screwing on the velocity measuring apparatus
s, Slots in a, leading into the interior of the pipe
au, b, t, Arrangement for attachment of a manometer

Figure 8.- Transition piece I.

Figure 10.- Determination of the
transition cross
sections.

Figure 9.- Transition piece II.
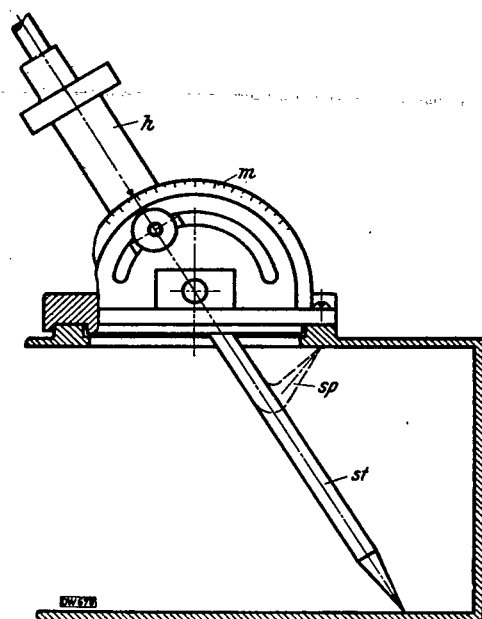
st, sp,   Caliper points
h,   Support
m,   Graduated circle
          (see also fig. 4)

Figure 11.- Caliper apparatus for
            measuring the trans-
ition cross section.

Figure 12.- Arrangement of
            measuring points
in sections 10 and 0

Cross section ⑩

y direction

x direction

Cross
section ⓪
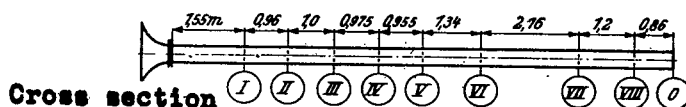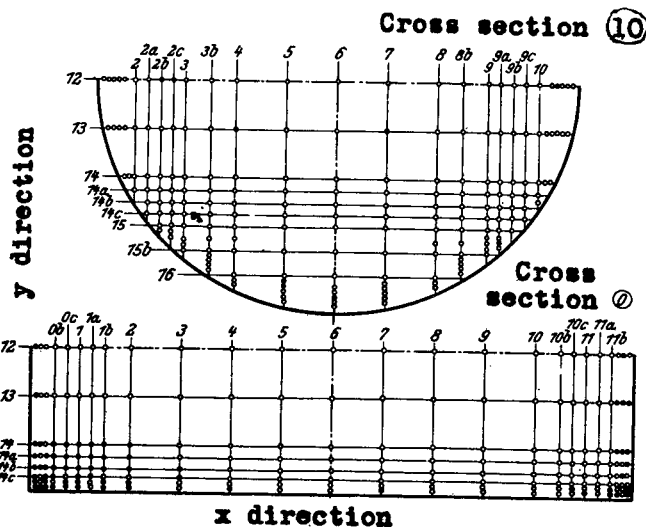
Cross section Ⓘ Ⓘ Ⓘ Ⓘ Ⓘ Ⓘ   Ⓘ Ⓘ Ⓘ

Figure 16.- Position of cross sections I to VIII and O
            in the rectangular entry run.
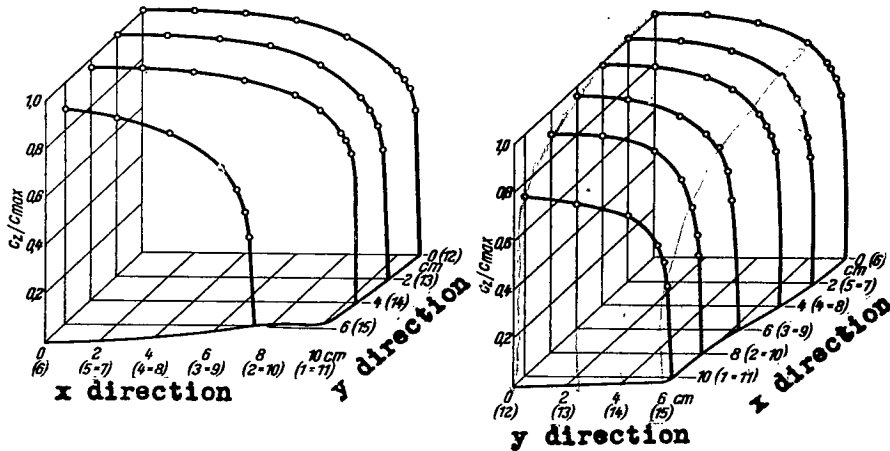
x direction   y direction
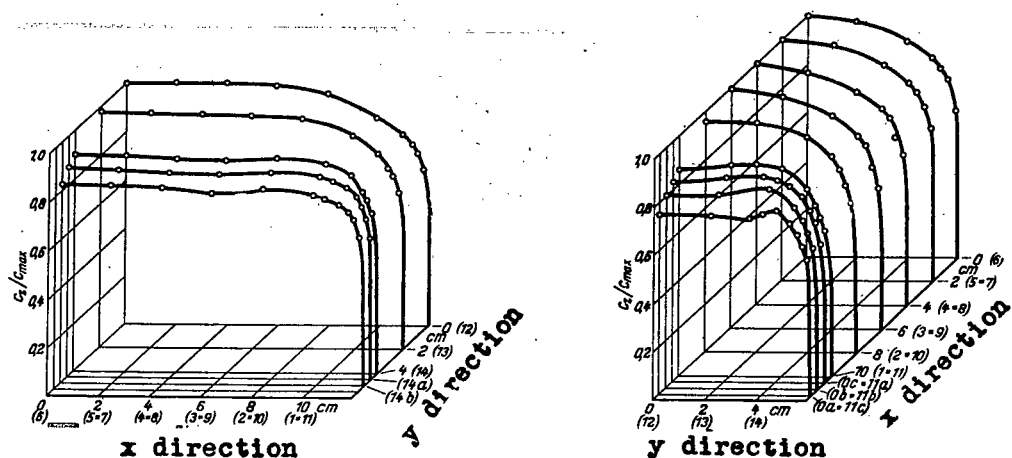
Figure 17.- Velocity distribution (c_z/c_{max}) in the horizontal (x) and
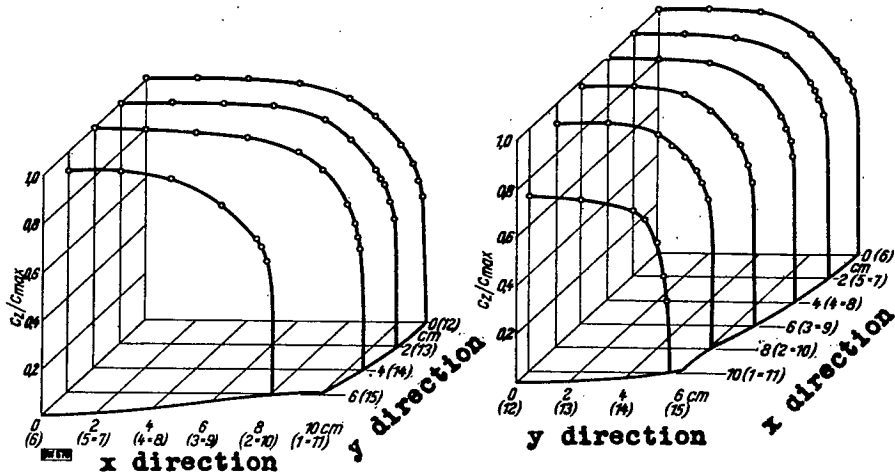   vertical (y) measuring planes in cross section O of the
transitions R⟶ C(I) and R⟶C(II). c_{max} = 19.5 m/s
(for the figures in parentheses see fig. 12).
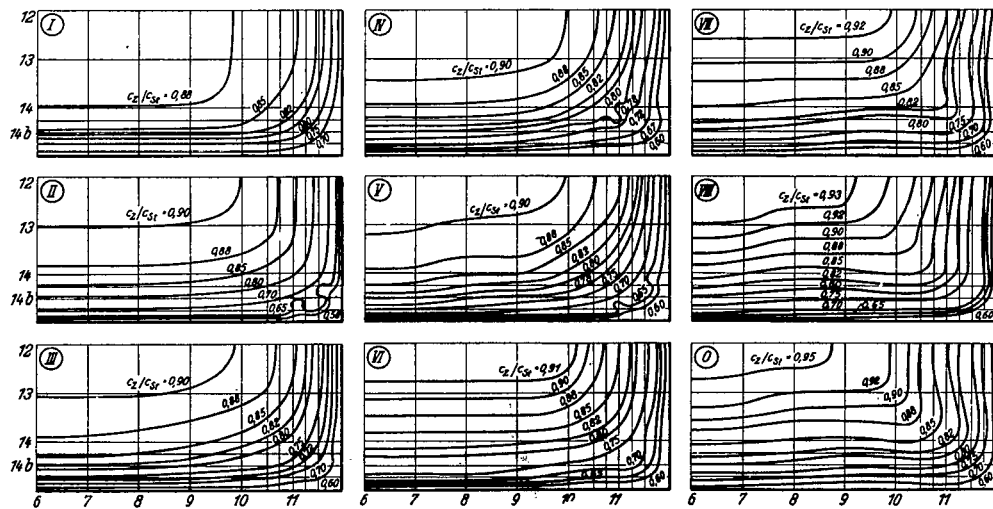
Figures 18 - 25.- Isotachs in cross   Figure 26.- Isotachs in
              sections I to VIII   cross section O
of the rectangular entry run.   of the transitions R ⟶ C
                                      (I) and R ⟶ C (II).
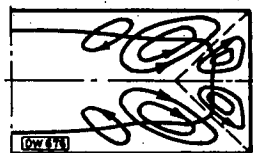
Figure 27.- Secondary flow
in a pipe of
rectangular section.
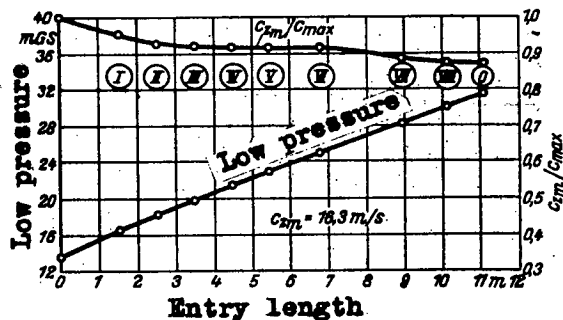
Figure 28.- c_{zm}/c_{max} and static
pressure as a function
of the entry length of the
rectangular pipe.
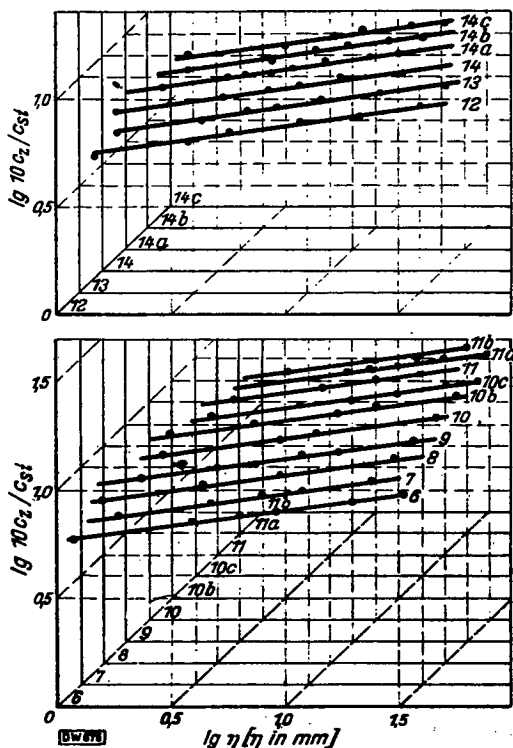
Figure 29.- Determination of the
exponent $1/n$ of the
velocity distribution in the
neighborhood of the wall in the
horizontal and vertical measuring
planes of cross section O for the
transitions R $\longrightarrow$ C(I) and R $\longrightarrow$ C(II).
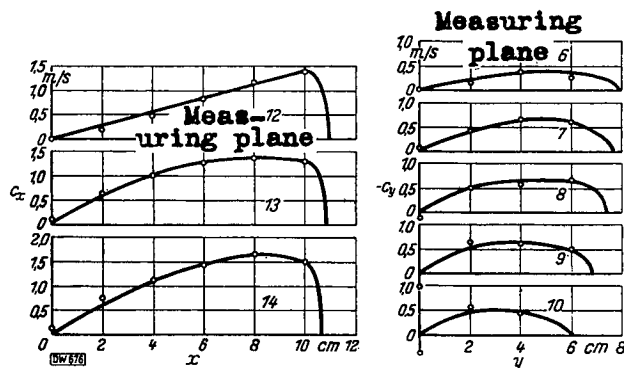
Figure 30.- c_x and c_y
distributions
in the horizontal and
vertical planes respectively
in cross section 5 of the
transition C $\longrightarrow$ R(I) .
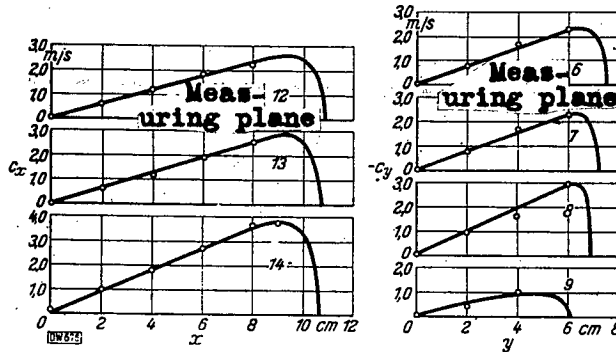
Figure 31.- c_x and c_y distributions in the horizontal and vertical measuring planes respectively for cross section 5 of the transition C $\longrightarrow$ R(II).

Figure 32.- Velocity distribution (c_z/c_{max}) in the horizontal (x) and vertical (y) measuring planes of cross section 5 of the transition C $\longrightarrow$ R(I).
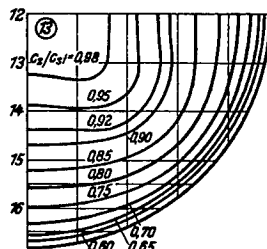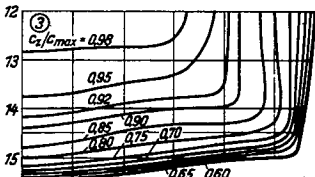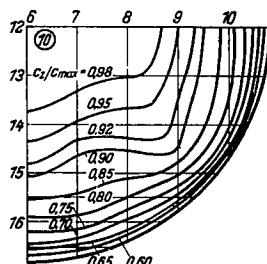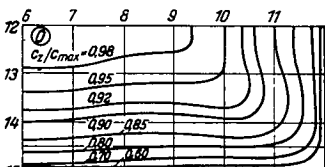

Figure 33.- Velocity distribution c_z/c_{max}) in the horizontal (x) and vertical (y) measuring planes of cross section 5 of the transition C $\longrightarrow$ R(II).
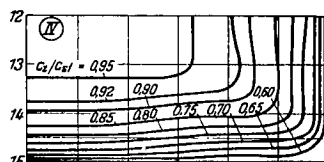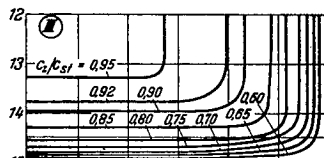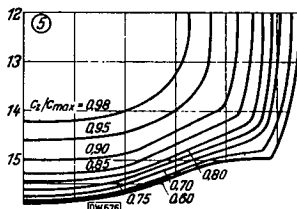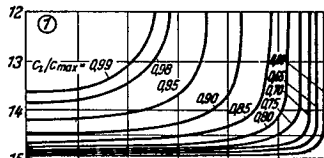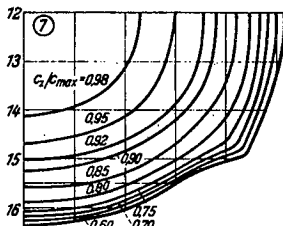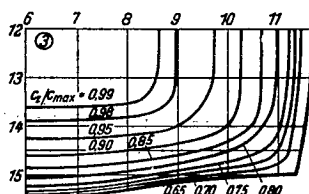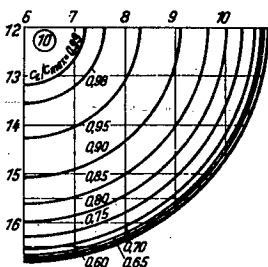
Figures 34 - 38.- Isotachs in cross sections 10,7,5,3 and 1 of the transition C → R(I). Fig. 34 is also for C → R(II).

Figures 39,40.- Isotachs in cross sections III and IV of exit run in the transition C → R (I).
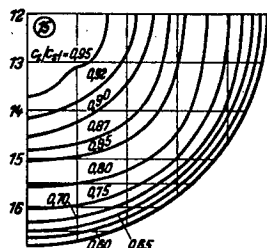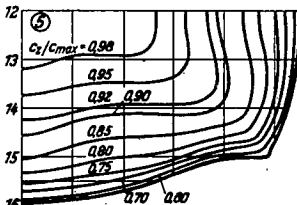
Figures 41-45.- Isotachs in cross sections 0,3,5,7 and 10 of the transition R → C (I). Fig. 41 also for R → C (II).

Figures 46,47.- Isotachs in cross sections 13,and 15 of exit run for the transition R → C (I).

Figures 48 - 51.- Isotachs in cross sections 9,5,1 and 0 of the transition C ⟶ R(II).

Figures 52 - 55.- Isotachs in cross sections 1,5,9 and 10 of transition R ⟶ C (II).

Figures 58 - 60.- Isobars in cross sections 9,5 and 1 of transition C ⟶ R (II).

Figures 56,57.- Isobars in cross sections 9 and 1 of the transition C ⟶ R (I).

———— Rectangular cross section O
    for transitions C⟶R(I)
    and C⟶R(II)
— — — Circular cross section 10
    for transitions C⟶R(I)
    and for C⟶R(II)

———— Rectangular cross section O
    for transitions R⟶C(I)
    and R⟶C(II)
— — — Circular cross section 10
    for transition R⟶C(I)
— · — Circular cross section 10
    for transition R⟶C(II)

Percent of cross section area F
$$c_{max} = 16.4\,m/s$$

Percent of cross section area F
$$c_{max} = 19.5\,m/s$$

Figures 61,62.- Minimum value of c_z/c_{max} plotted against percent
cross section area for sections O and 10.

Transition C⟶R(I) $c_{zm} = 14.0\,m/s$

Transition R⟶C(I) $c_{zm} = 16.3\,m/s$

Transition C⟶R(II)

Transition R⟶C(II) $c_{zm} = 14.0\,m/s$, $c_{zm} = 16.3\,m/s$

a, Entry run
v, Transition piece
b, Exit run
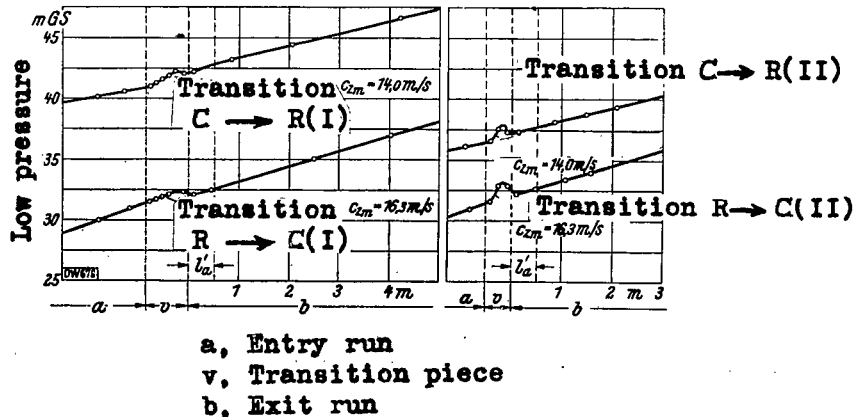
Figures 63,64.- Variation of pressure at the wall along the test
length for the various transitions.